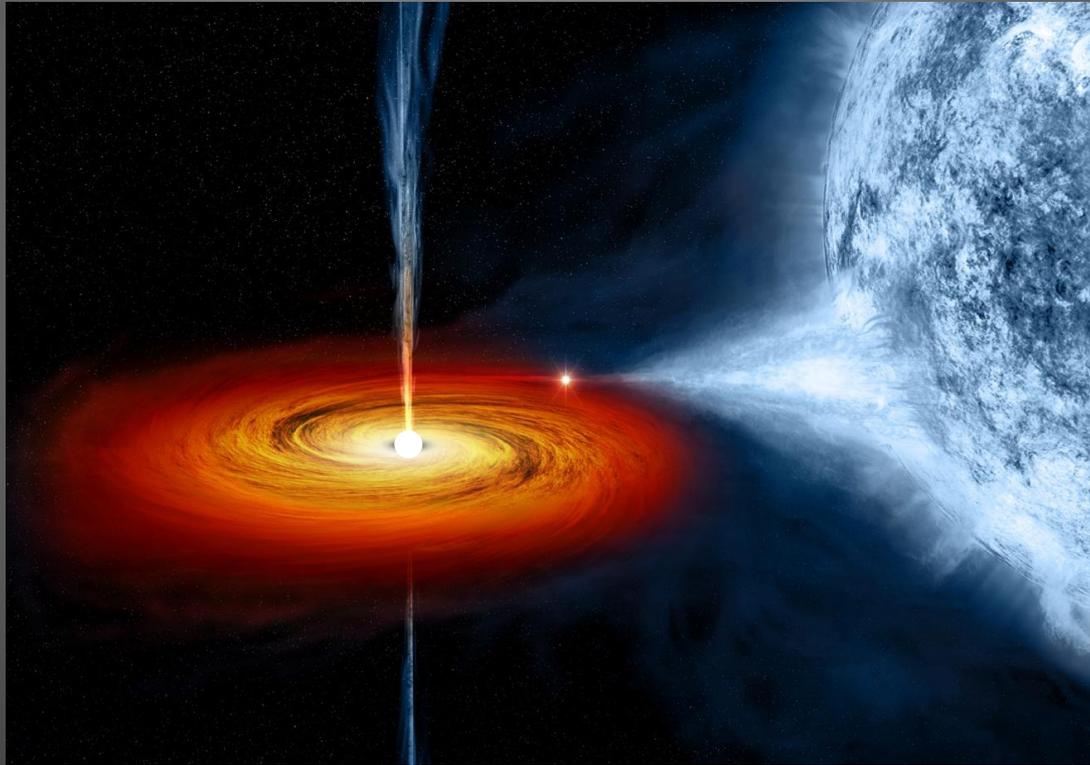


# X-RAY BINARIES AND RECYCLING OF MILLISECOND PULSARS

AEI IMPRS GW LECTURES 1+2



Thomas Tauris – IFA, Aarhus University

**Note: These lectures will be recorded and posted onto the IMPRS website**

Dear participants,

We will record all lectures on “The Astrophysics of Compact Objects”, including possible Q&A after the presentation, and we will make the recordings publicly available on the IMPRS lecture website at:

<https://imprs-gw-lectures.aei.mpg.de>

By participating in this Zoom meeting, you are giving your explicit consent to the recording of the lecture and the publication of the recording on the course website.

# Albert-Einstein Institute Lectures 2021

Thomas Tauris @ Aarhus University



Lectures 1+2: [Wednesday May 12, 10:00 – 12:00](#)

**X-ray Binaries and Recycling Millisecond Pulsars**

Lectures 3+4: [Friday May 14, 10:00 – 12:00](#)

**Spin and B-field Evolution of Neutron Stars (+ Black Hole Spins)**

Lectures 5+6: [Wednesday May 19, 10:00 – 12:00](#)

**Formation of Binary Neutron Stars/Black Holes**

Lectures 7+8: [Friday May 21, 10:00 – 12:00](#)

**Binary Neutron Stars and Gravitational Waves at Low and High Frequencies**

You are most welcome to ask questions any time 😊

# X-RAY BINARIES AND RECYCLING OF MILLISECOND PULSARS

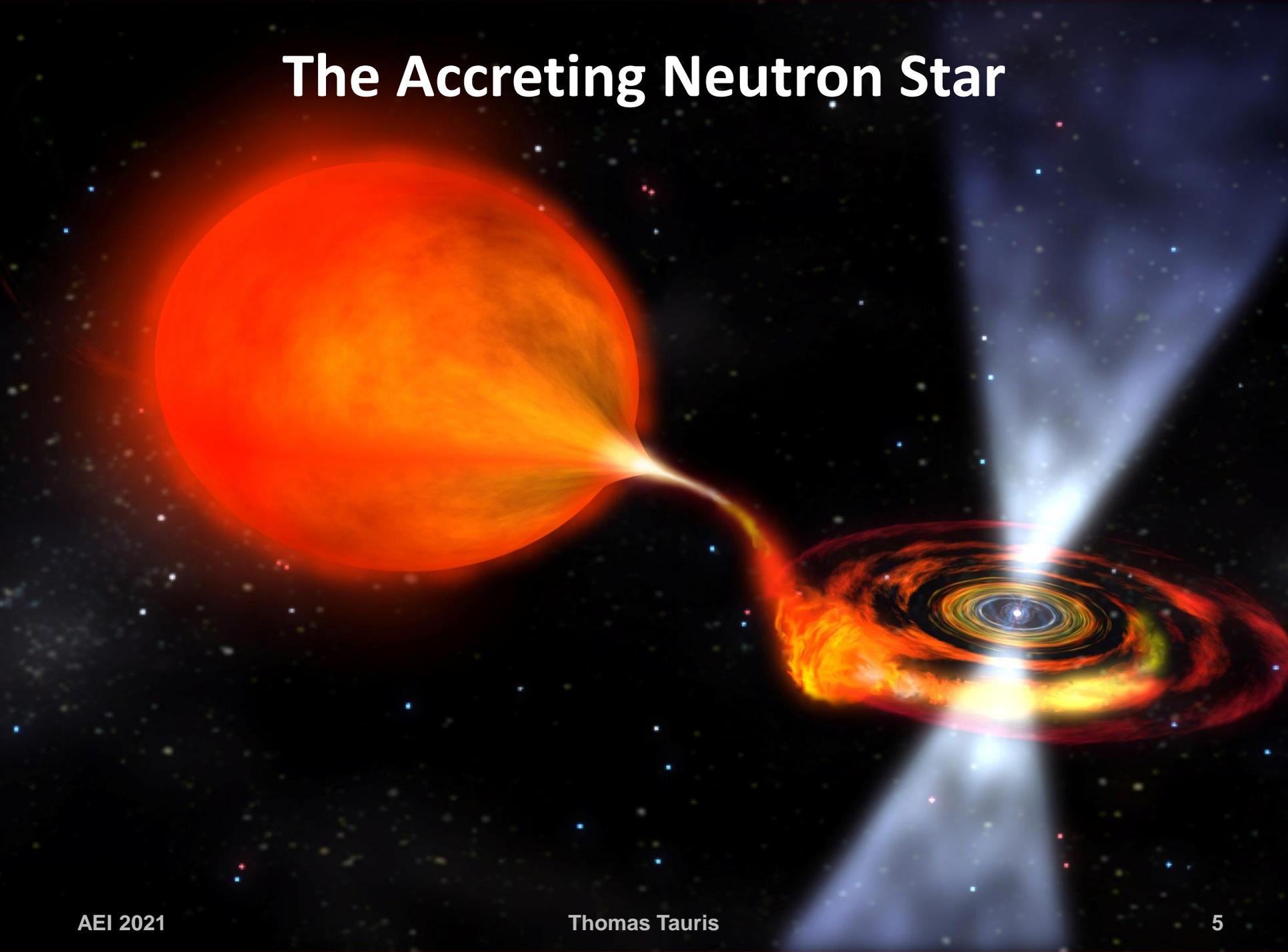
## AEI LECTURES 1+2

- Introduction to X-ray binaries/Accretion
  - HMXBs and LMXBs
  - Roche-lobe overflow - Cases A, B and C
  - Stability criteria for mass transfer / stellar evolution
  - Orbital angular momentum balance equation
- Common envelope and spiral-in evolution
- Equilibrium spin period and spin-up line in  $P$ - $\dot{P}$  diagram
- Accretion physics
  - Accretion disks
  - B-field decay
  - Four phases of accretion (self study)

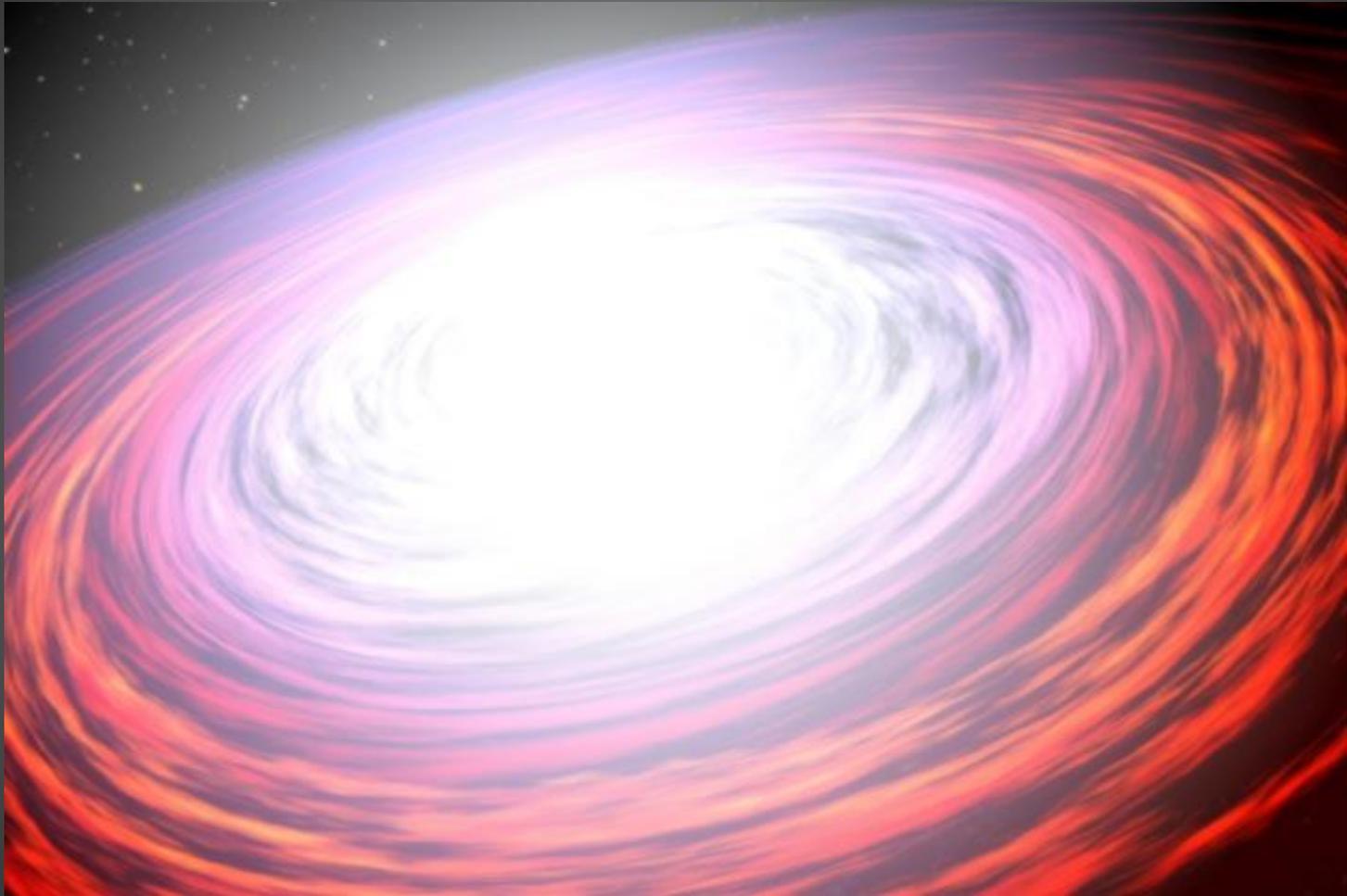
For a review: Tauris & van den Heuvel (2006)  
Tauris & van den Heuvel (2022)  
New textbook from Princeton Uni. Press



# The Accreting Neutron Star



# Energetics of Accretion



# DISCOVERY OF BLACK HOLES AND NEUTRON STARS

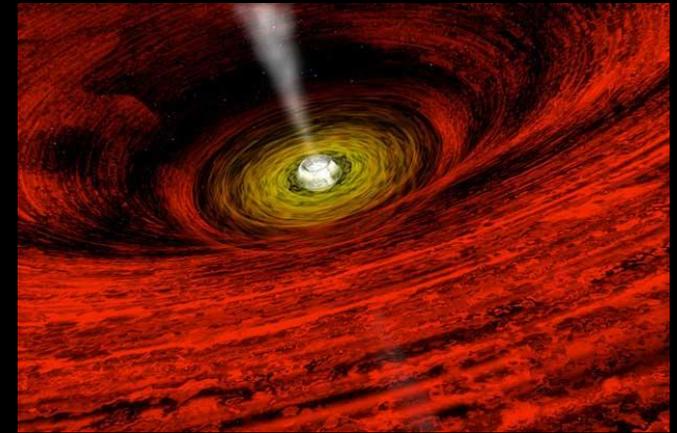


**1962.** Sco X-1: neutron star LMXB  
(Giacconi, Aerobee 150 rocket)

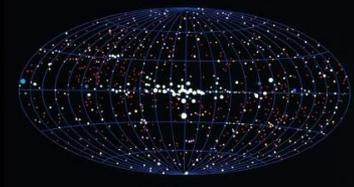
**1967.** PSR B1919+21

**1971.** Cen X-3: pulses (P=4.84 sec)

**1971.** Cyg X-1: black hole HMXB



$$1 M_{\odot} \text{ accretor} \quad \text{and} \quad L_x = 10^{37} \text{ erg s}^{-1}$$



Stellar object	Radius (km)	$\Delta U/mc^2$	$\Delta U/m$ (erg g <sup>-1</sup> )	$dM/dt$ (M <sub>sun</sub> yr <sup>-1</sup> )	Column density (g cm <sup>-2</sup> )
Sun / star	$7 \times 10^5$	$2 \times 10^{-6}$	$2 \times 10^{15}$	$1 \times 10^{-4}$	140  
White dwarf	10000	$2 \times 10^{-4}$	$1 \times 10^{17}$	$1 \times 10^{-6}$	16  
Neutron star	10	0.15	$1 \times 10^{20}$	$1 \times 10^{-9}$	0.5  
Black hole	3	0.1–0.4	$4 \times 10^{20}$	$4 \times 10^{-10}$	0.3  

 Note: X-rays are stopped at column densities larger than a few g cm<sup>-2</sup>

# Accretion Luminosity

$$L_X = (\varepsilon_{nuc} + \varepsilon_{grav}) \dot{M}_{acc}$$

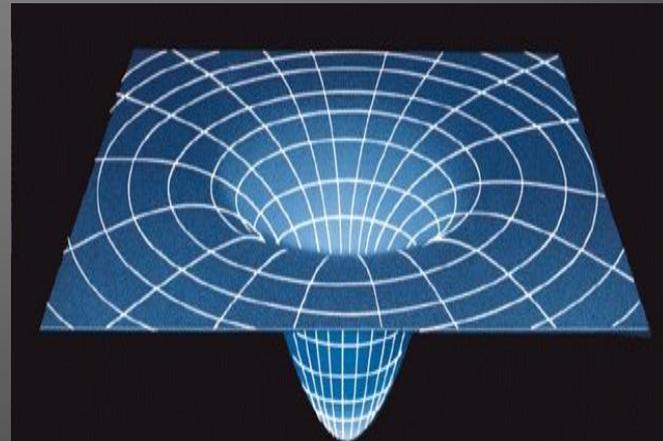
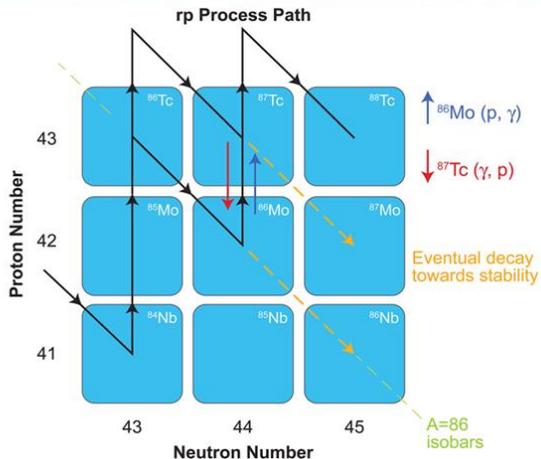
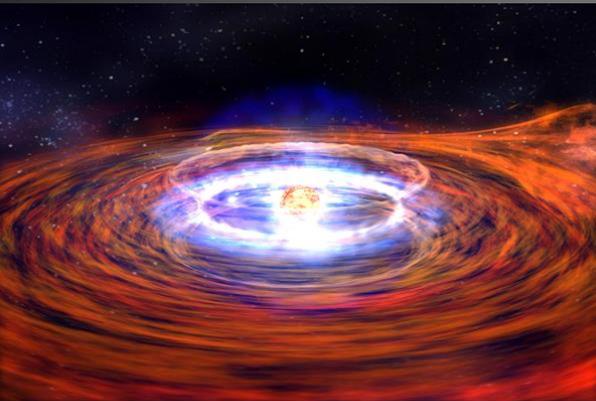
Nuclear burning at surface of compact object



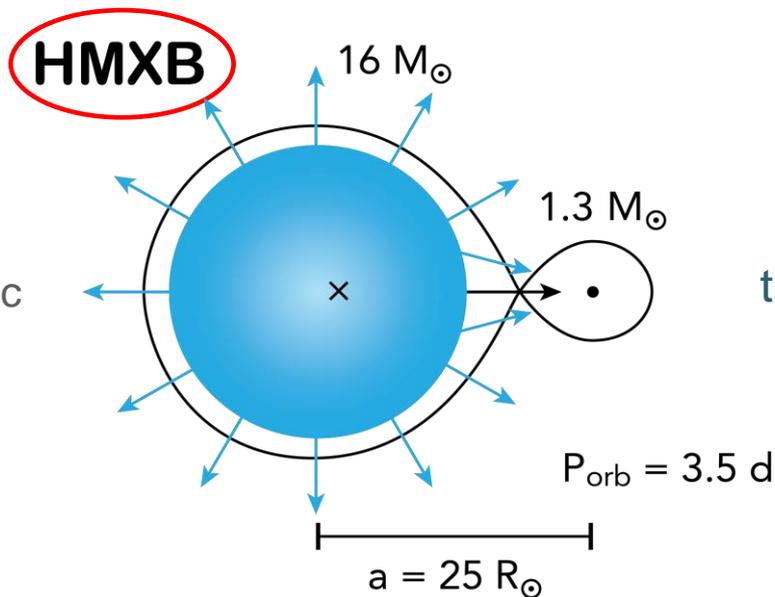
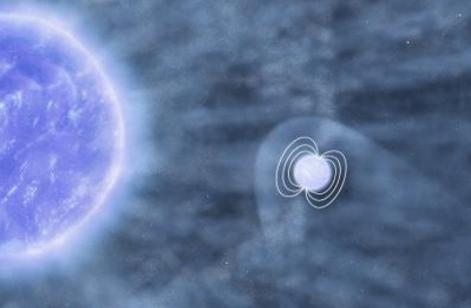
Dominate in accreting WD  
(not relevant for BH)

Dominate in NS and BH

Release of gravitational binding energy



# High-Mass X-ray Binaries



- wind accretion
- beginning atmospheric Roche-lobe overflow

timescale:  $10^5 - 10^6 \text{ yr}$

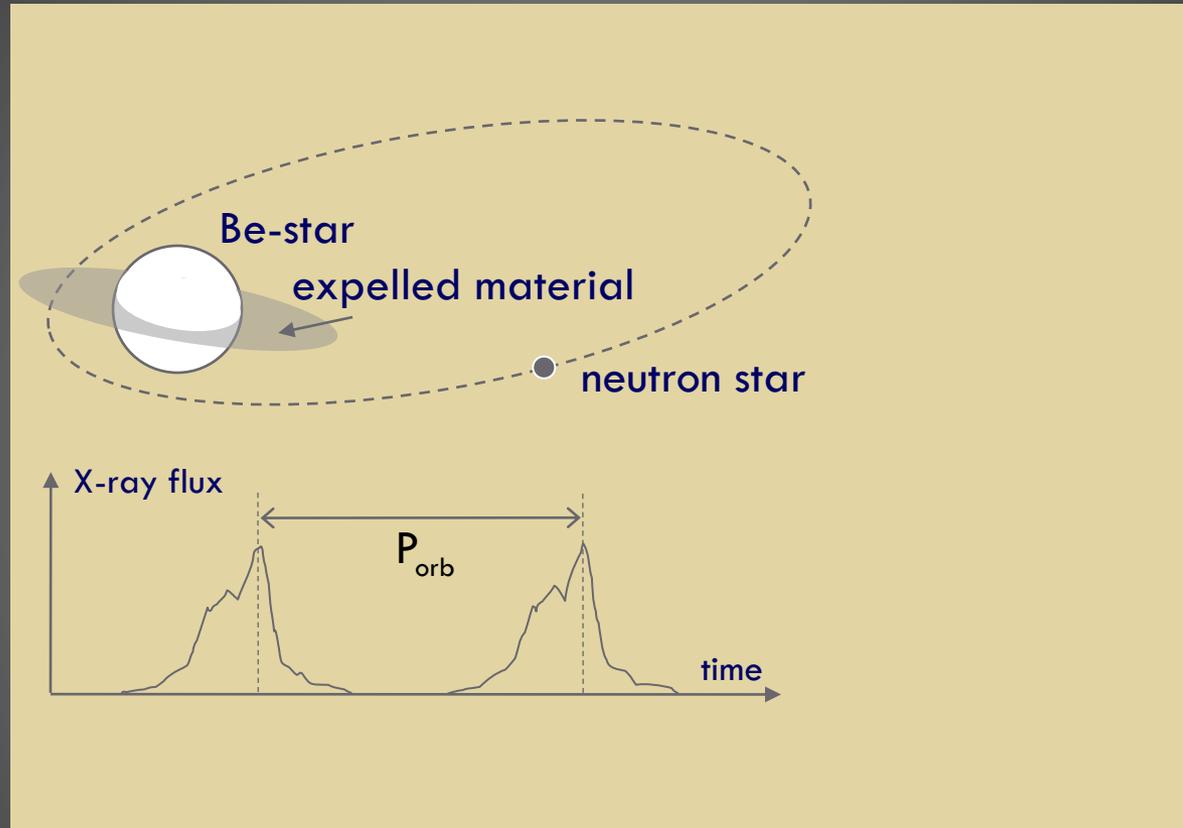
Donor star masses  $M_2 > 10 M_{\text{sun}}$

114 systems known in our Galaxy:

- 50 systems with measured  $P_{\text{orb}} = 0.2 - 262 \text{ days}$  (median 23 days)
- 66 systems with measured  $P_{\text{spin}} = 33 \text{ ms} - 4 \text{ hours}$  (median 3 min)

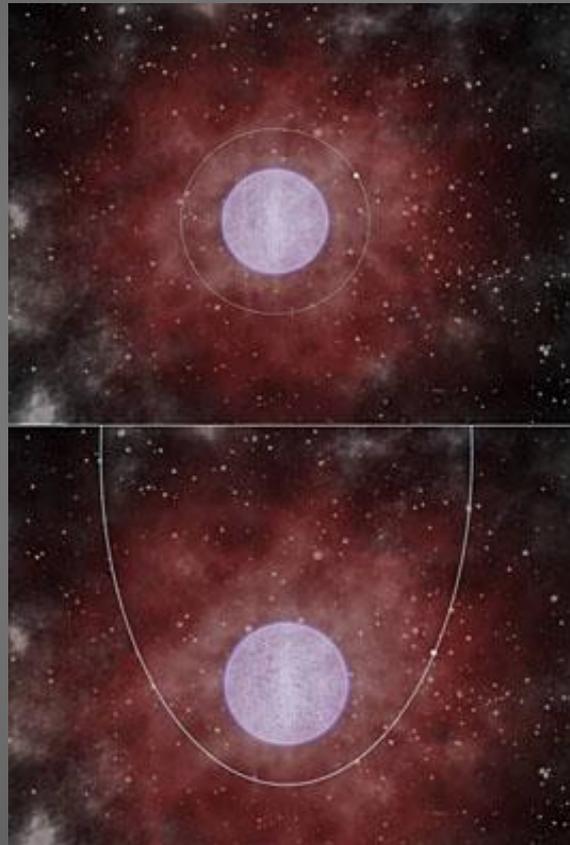
# Be-star X-ray Binaries

(subclass of HMXBs)



# Supergiant Fast X-ray Transients (SFXTs)

(subclass of HMXBs – many discovered with INTEGRAL)



# HMXBs with BH accretors

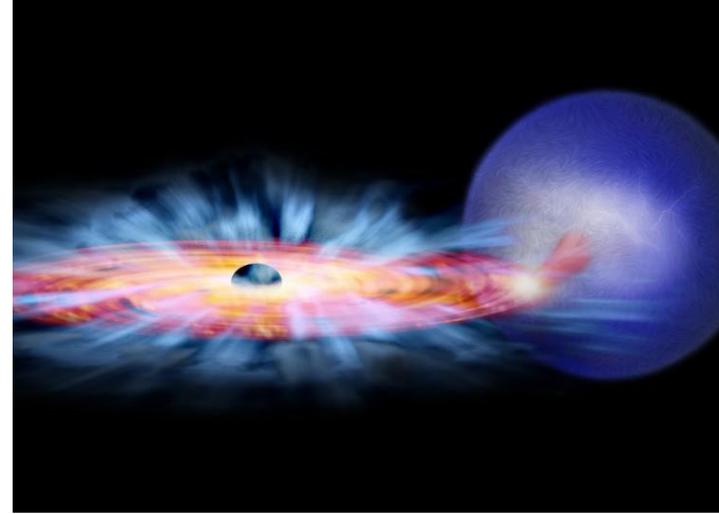


Table 6.6: The five known BH HMXBs. (Updated after [van den Heuvel, 2019](#)).

Source	$P_{\text{orb}}$ (d)	$M_{\text{donor}}$ ( $M_{\odot}$ )	$M_{\text{BH}}$ ( $M_{\odot}$ )	Reference
Cyg X-1	5.6	41 ( $\pm 7$ )	21.2 ( $\pm 2.2$ )	<a href="#">Miller-Jones et al. (2021)</a>
LMC X-1	3.9	31.8 ( $\pm 3.5$ )	10.9 ( $\pm 1.4$ )	<a href="#">Orosz et al. (2009)</a>
LMC X-3	1.7	3.6 ( $\pm 0.6$ )	7.0 ( $\pm 0.6$ )	<a href="#">Orosz et al. (2014)</a>
MCW 656	$\sim 60$	$\sim 13$	4.7 ( $\pm 0.9$ )	<a href="#">Casares et al. (2014)</a>
M33 X-7	3.45	70 ( $\pm 7$ )	15.7 ( $\pm 1.5$ )	<a href="#">Orosz et al. (2007)</a>

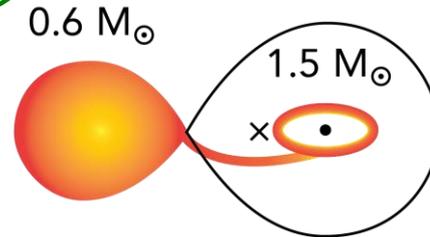
Tauris & van den Heuvel (2022)

# Low-Mass X-ray Binaries



LMXB

- Roche-lobe overflow



timescale:  $10^8 - 10^9$  yr

$a = 1.9 R_{\odot}$   $P_{\text{orb}} = 0.21$  d

Donor star masses  $M_2 < 1 M_{\text{sun}}$

187 candidate systems in our Galaxy:

- 74 systems with measured  $P_{\text{orb}} = 11$  min – 1160 days (median: 8 hours)

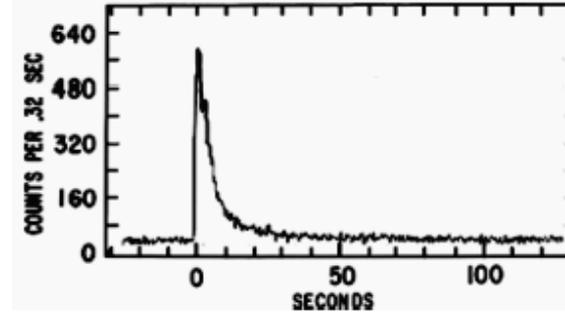
- 26 systems with measured  $P_{\text{spin}} = 1.6$  ms – 7.7 sec (median: 3 ms)

Most systems are **transients** and **X-ray bursters** (thermonuclear explosions)

About 20 **black hole** systems (**soft X-ray transients**)

# X-ray Bursts

Burst light curve



Rise time  $\approx 0.5 - 5$  seconds  
Decay time  $\approx 10 - 100$  seconds  
Recurrence time  $\approx$  hours to day  
Energy release  $\approx 10^{39}$  erg

Accretion of H-rich matter piles up on the NS. Gravity compresses matter and the temperature rises at the base of this accumulated envelope leading to ignition under degenerate conditions resulting in a thermonuclear explosion and a rapid photospheric expansion.

The burst lasts for some 10-100 sec.

Burst oscillations reveal the spin period of the NS.

HARDY

# LMXBs with BH accretors

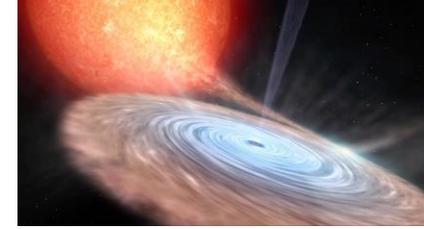
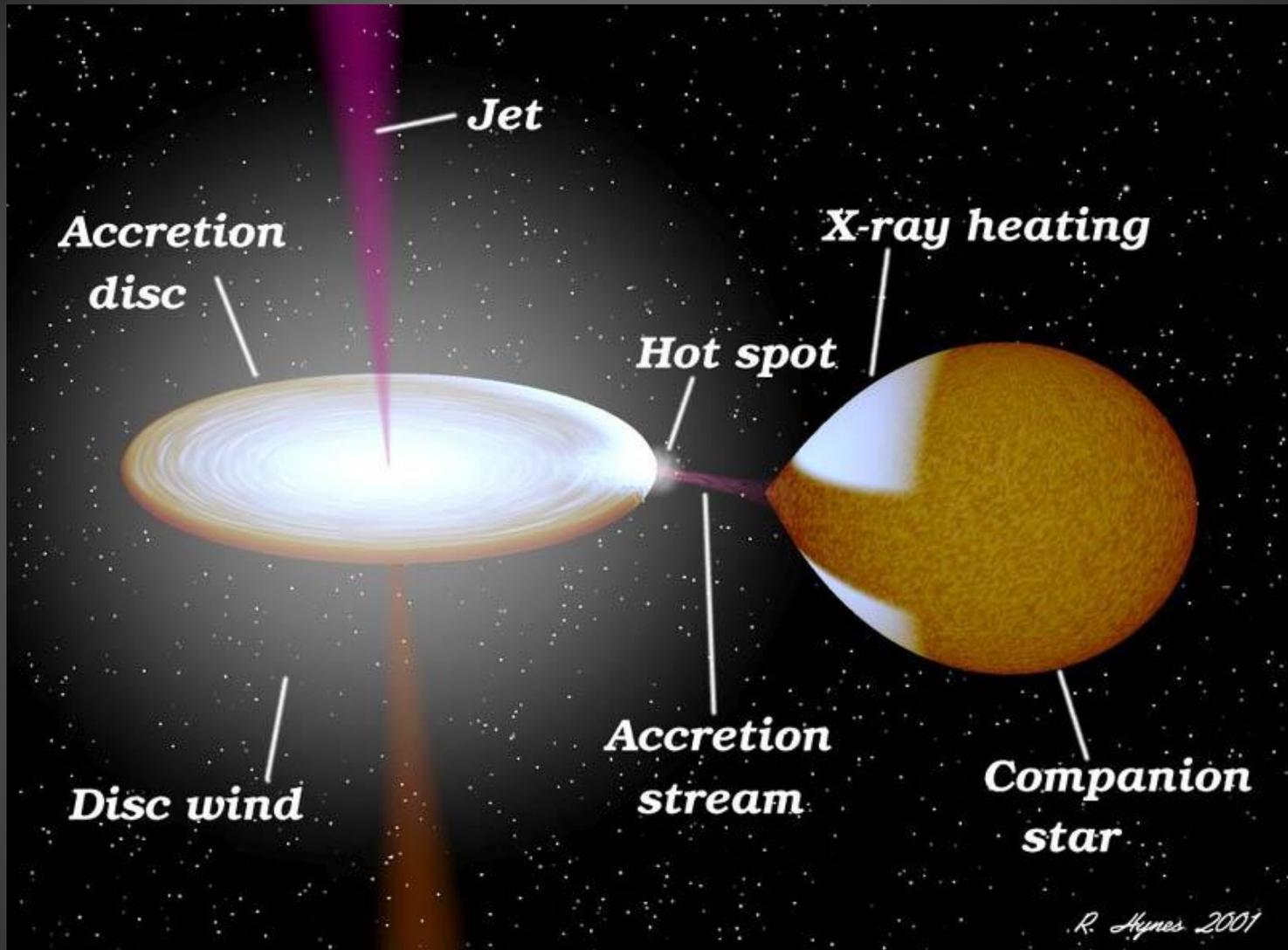


Table 6.5: Examples BH LMXBs. (After [McClintock & Remillard, 2006](#)).

Source	Alternative name	$P_{\text{orb}}$ (hr)	Spectrum	BH mass ( $M_{\odot}$ )
0422+32	V518 Per	5.1	M2V	3.2 – 13.2
0620–003	V616 Mon	7.8	K4V	3.3 – 12.9
1009–45	MM Vel	6.8	K7/M0V	6.3 – 8.0
1118+480	KV UMa	4.1	K5/M0V	6.5 – 7.2
1124–684	GU Mus	10.4	K3/K5V	6.5 – 8.2
1543–475	IL Lupi	26.8	A2V	7.4 – 11.4
1550–564	V381 Nor	37.0	G8–K8IV	8.4 – 10.8
1655–40	V1033 Sco	62.9	F3–F5IV	6.0 – 6.6
1659–487	V821 Ara	42.1	—	—
1705–250	V2107 Oph	12.5	K3/7V	5.6 – 8.3
1819.3–2525	V4641 Sgr	67.6	B9III	6.8 – 7.4
1859+226	V406 Vul	9.2	—	7.6 – 12.0
1915+105	V1487 Aql	804.0	K/MIII	10.0 – 18.0
2000+251	QZ Vul	8.3	K3/K7V	7.1 – 7.8
2023+338	V404 Cyg	155.3	K0III	10.1 – 13.4

Tauris & van den Heuvel (2022)



# Equipotential Surfaces and RLO

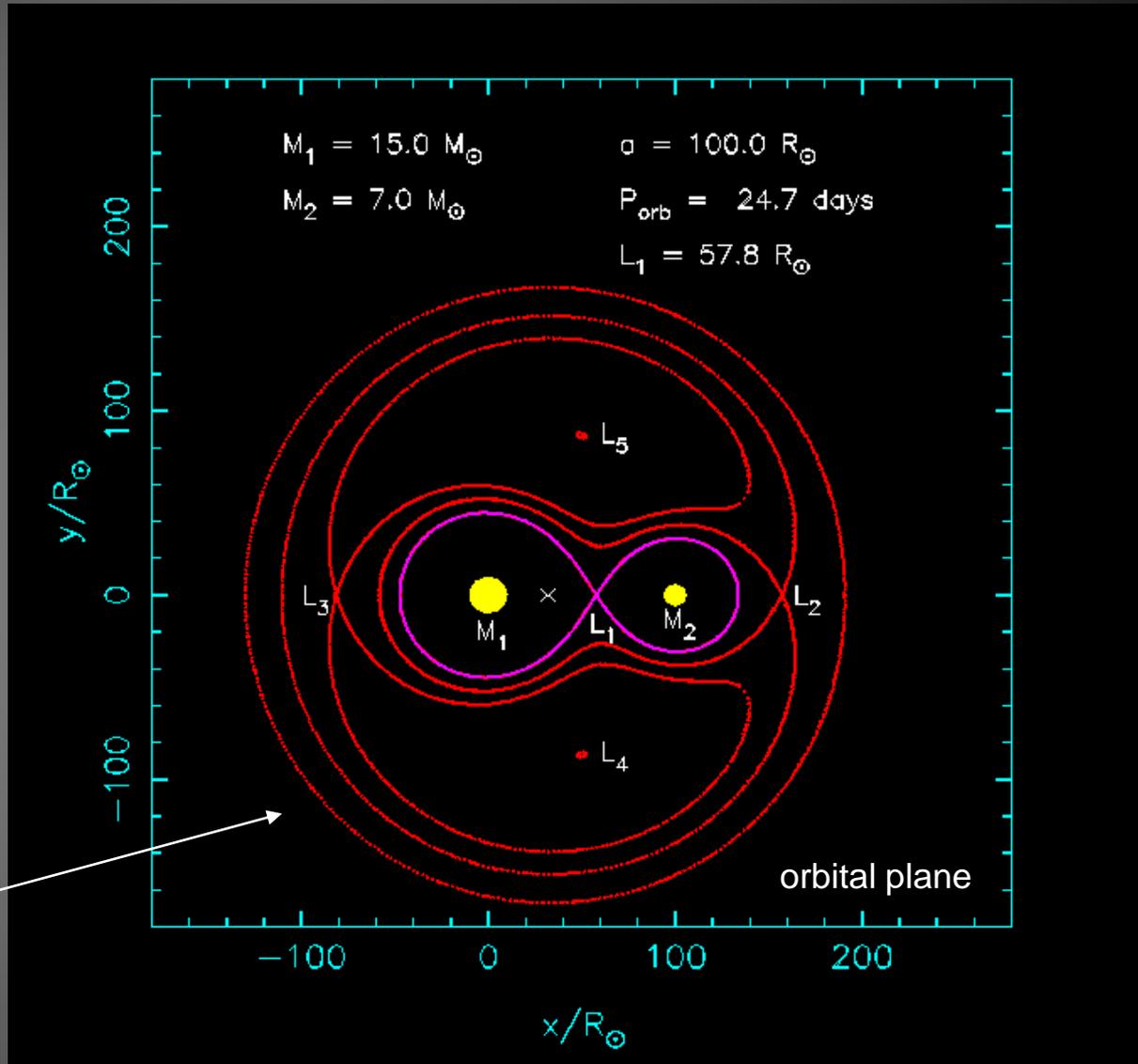
Effective gravitational potential:

$$\Phi = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{\Omega^2 r_3^2}{2}$$

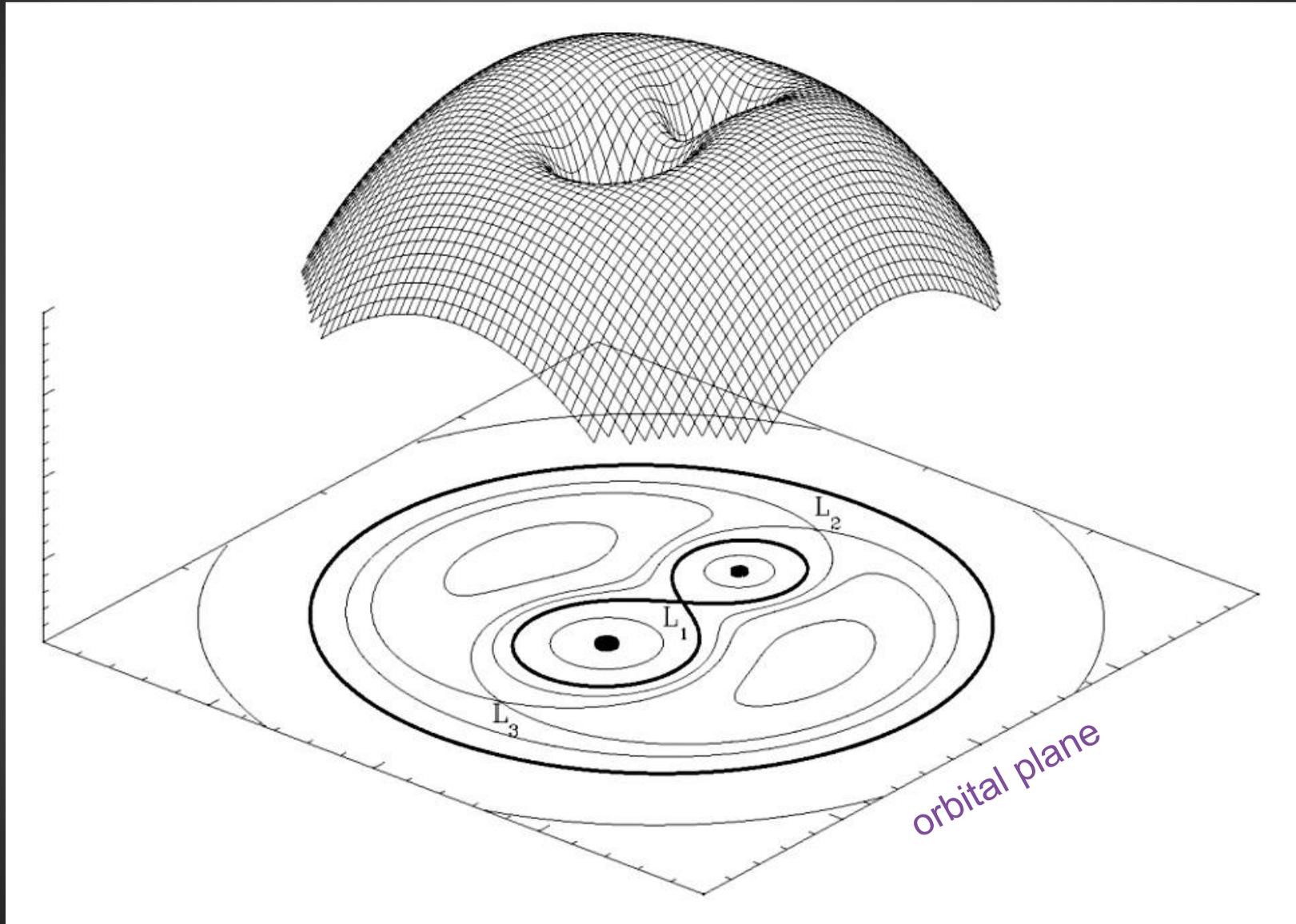
Roche-lobe radius:

$$\frac{R_L}{a} = \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})}$$

co-moving frame



# Equipotential Surfaces and RLO



# X-ray binaries Roche-lobe overflow Cases A, B and C:

(the evolutionary stage of the donor star at onset of RLO is quite important ...)

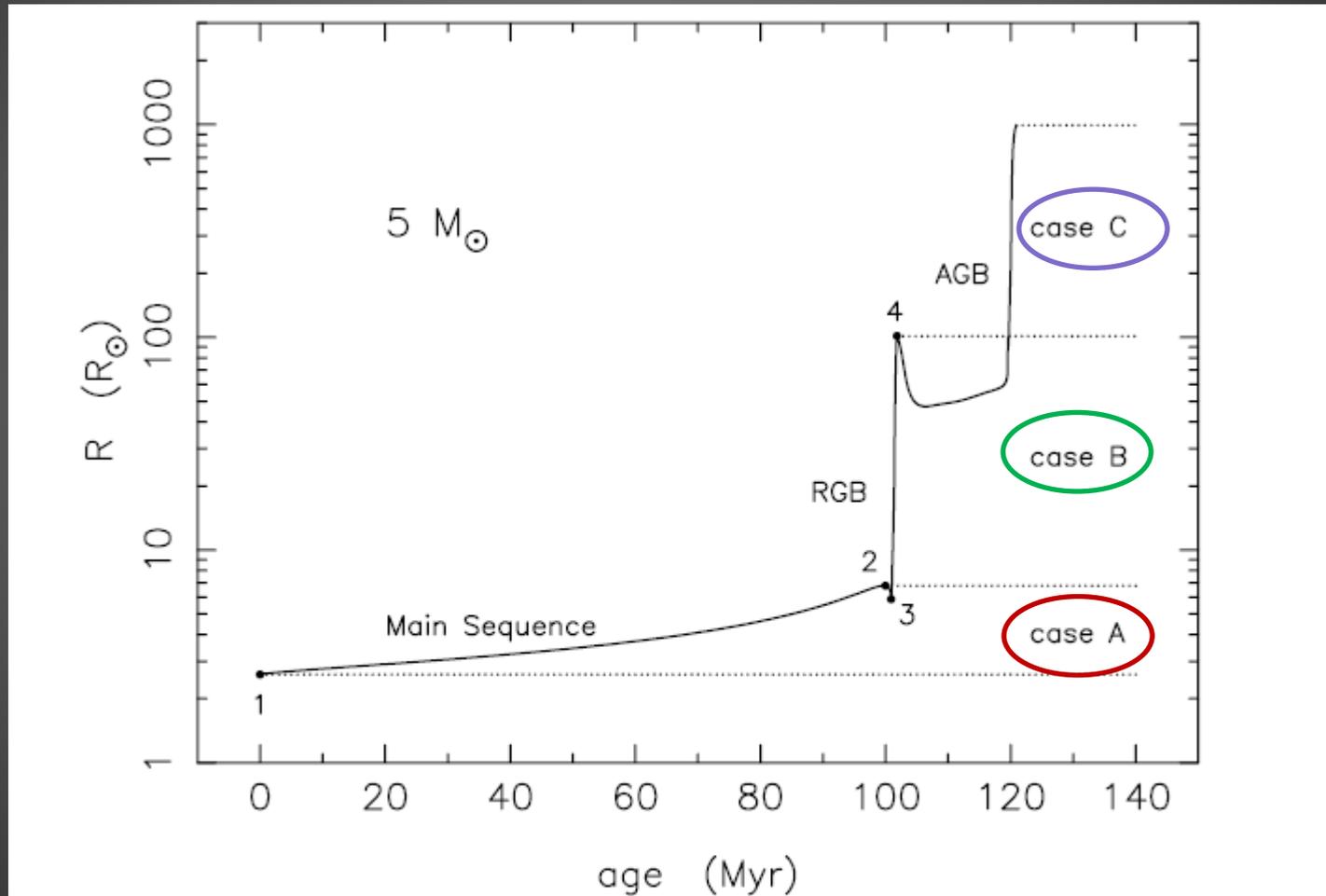


Fig. 16.6. Evolutionary change of the radius of the  $5 M_{\odot}$  star plotted in Fig. 16.5. The ranges of radii for mass transfer to a companion star in a binary system according to RLO cases A, B and C are indicated – see Section 16.4 for an explanation.

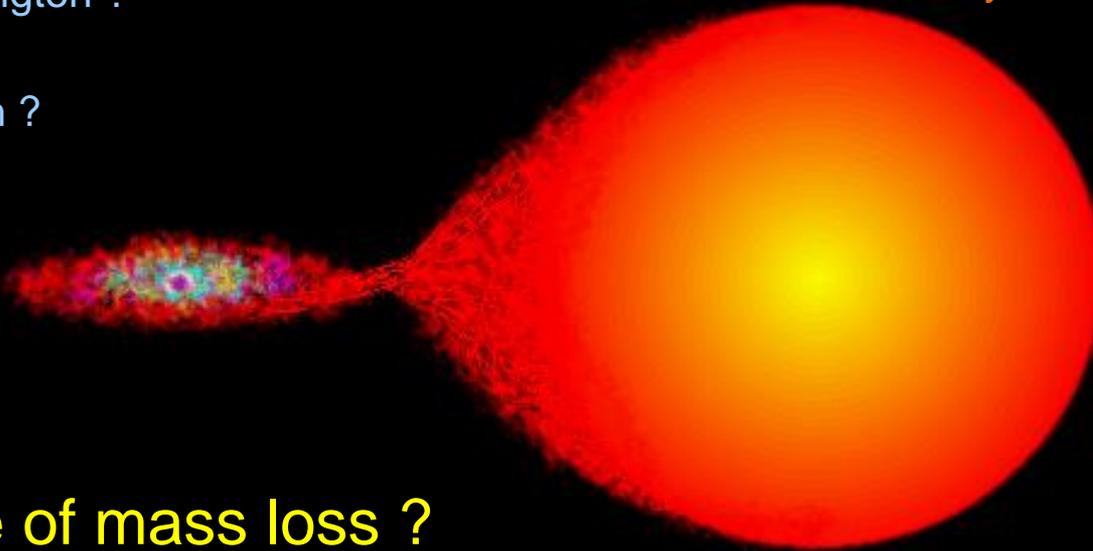
# The evolution of compact binaries

## Accretion ?

super-Eddington ?  
jet ?  
B-field, spin ?

## Stability ?

response of donor star ?  
response of Roche-lobe ?  
dynamically stable ?



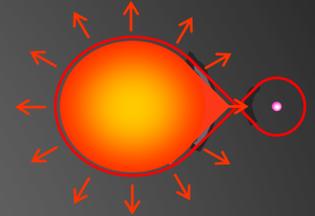
## Mode of mass loss ?

specific orbital angular momentum ?

magnetic braking / tidal interactions  
gravitational wave radiation

# Stability Criteria for Mass Transfer

exponents of radius to mass:  $R \propto M^\zeta$



$$\zeta_{donor} \equiv \frac{\partial \ln R_2}{\partial \ln M_2} \quad \wedge \quad \zeta_L \equiv \frac{\partial \ln R_L}{\partial \ln M_2}$$

adiabatic or thermal response of the donor star to mass loss

initial stability criteria:  $\zeta_L \leq \zeta_{donor}$

$$\dot{R}_2 = \left. \frac{\partial R_2}{\partial t} \right|_{M_2} + R_2 \zeta_{donor} \frac{\dot{M}_2}{M_2}$$

nuclear burning

$$\dot{R}_L = \left. \frac{\partial R_L}{\partial t} \right|_{M_2} + R_L \zeta_L \frac{\dot{M}_2}{M_2}$$

tidal spin-orbit couplings  
gravitational wave radiation

$$\dot{R}_2 = \dot{R}_L \quad \text{yields mass loss rate!}$$

# Stability Criteria for Mass Transfer. II

## Isotropic Re-emission Model

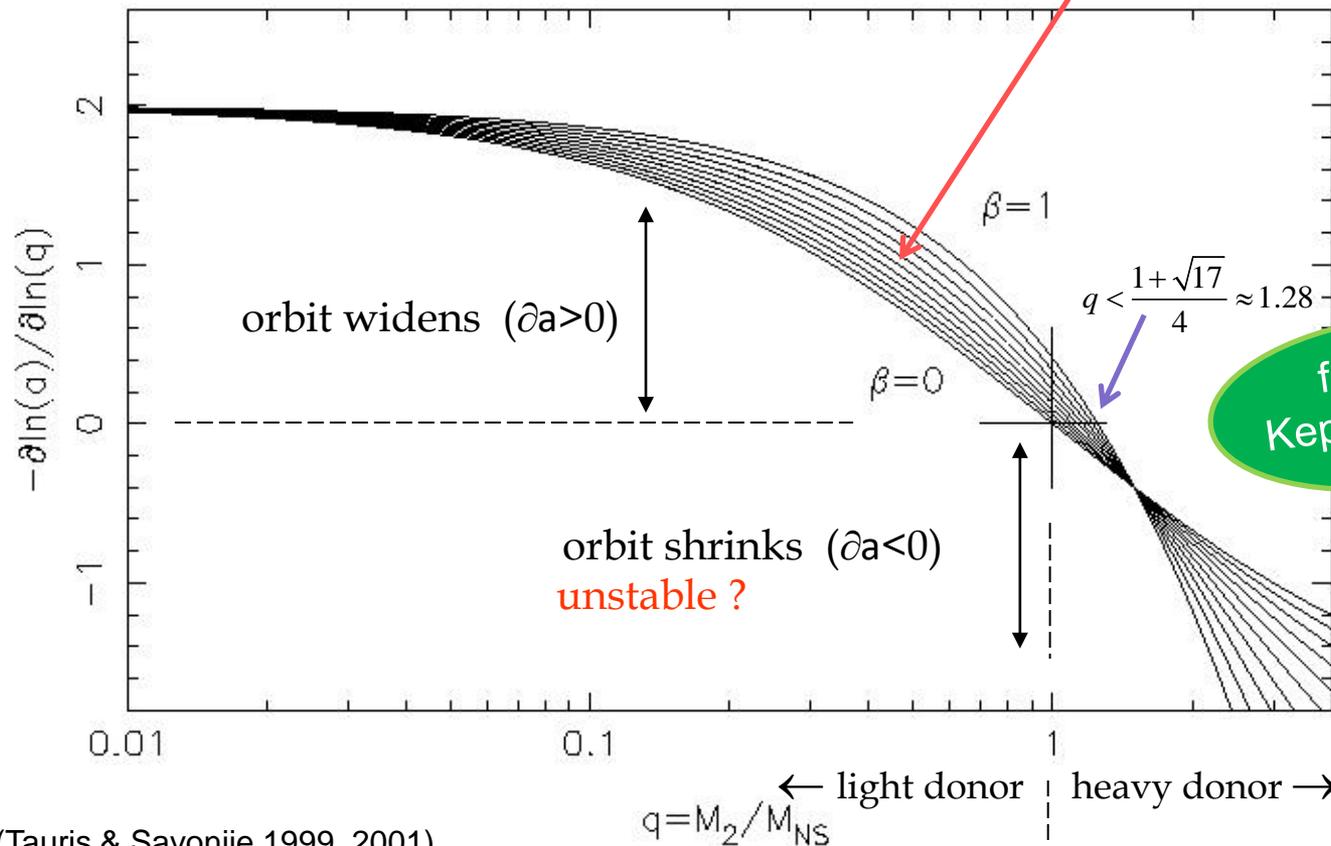
(Bhattacharya & van den Heuvel 1991)

(Soberman, Phinney & van den Heuvel 1997)

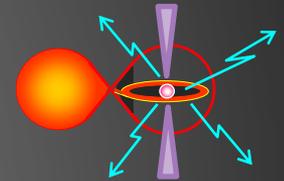
Orbital evolution:

$$-\frac{\partial \ln a}{\partial \ln q} \wedge q = \frac{M_2}{M_{NS}} \quad (\partial q < 0)$$

$$\beta = \max\left(\frac{|\dot{M}_2| - \dot{M}_{Edd}}{|\dot{M}_2|}, 0\right) \quad \alpha = 0 \quad \delta = 0$$



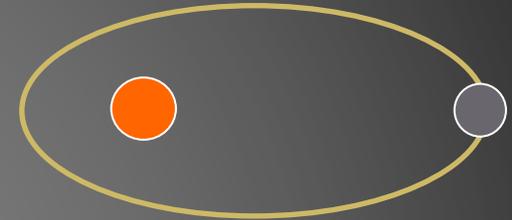
(Tauris & Savonije 1999, 2001)



# Orbital Angular Momentum Balance (OAMB) Eqn.

$$J_{orb} = \frac{M_1 M_2}{M} \Omega a^2 \sqrt{1-e^2}$$

orbital angular momentum



logarithmic differentiation  
( $e=0$ , tidal circularization)



$$\frac{\dot{a}}{a} = 2 \frac{\dot{J}_{orb}}{J_{orb}} - 2 \frac{\dot{M}_1}{M_1} - 2 \frac{\dot{M}_2}{M_2} + \frac{\dot{M}_1 + \dot{M}_2}{M}$$

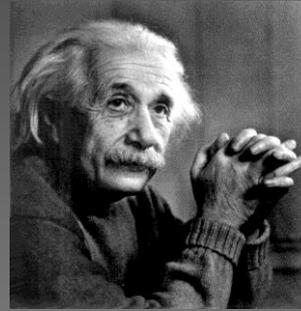
$$J_{orb} = |\vec{r} \times \vec{p}|$$

$$\Omega = \sqrt{GM / a^3}$$

$$\frac{\dot{J}_{orb}}{J_{orb}} = \underbrace{\frac{\dot{J}_{gwr}}{J_{orb}} + \frac{\dot{J}_{mb}}{J_{orb}} + \frac{\dot{J}_{ls}}{J_{orb}} + \frac{\dot{J}_{ml}}{J_{orb}}}_{\text{See next two slides}}$$

See next two slides

## Gravitational Wave Radiation:



$$\frac{\dot{J}_{gwr}}{J_{orb}} = - \frac{32}{5} \frac{G^3}{c^5} \frac{M_1 M_2 M}{a^4}$$

## Magnetic braking:

$$\frac{\dot{J}_{mb}}{J_{orb}} \propto - \frac{k^2 R^4}{a^5} \frac{GM^3}{M_1 M_2}$$

(uncertain)

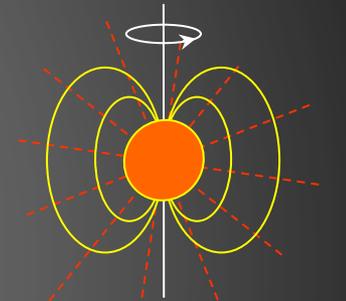
**Low-mass stars:** magnetic wind!

⇒ loss of spin angular momentum

In tight binaries the system is tidally locked (synchronized)

and spin-orbit couplings operate

⇒ loss of orbital angular momentum



$$M_2 < 1.5 M_{\text{sun}}$$

$$P_{\text{orb}} < 2 \text{ days}$$

## Spin-orbit couplings:

$$\frac{\dot{J}_{ls}}{J_{orb}}$$

- fx change in stellar moment of inertia

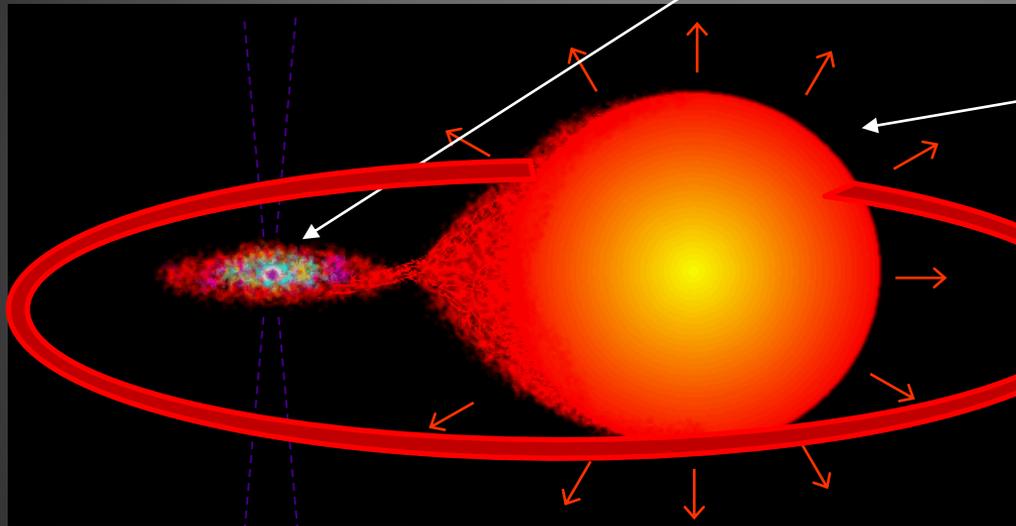
(as a result of nuclear burning or mass loss)

## Mass loss: (isotropic re-emission model)

$$\beta = \max\left(\frac{|\dot{M}_2| - \dot{M}_{Edd}}{|\dot{M}_2|}, 0\right)$$

$$\frac{\dot{J}_{ml}}{J_{orb}} = \frac{\alpha + \beta q^2 + \delta \gamma (1+q)^2}{1+q} \frac{\dot{M}_2}{M_2}$$

$\beta$ : mass ejected from accretor  
 - fx. in a relativistic jet  
 (isotropic re-emission)



$\alpha$ : direct fast wind

$\delta$ : mass loss via circumbinary coplanar toroid with radius:  $\gamma^2 a$

Accretion efficiency:  $\varepsilon = 1 - \alpha - \beta - \delta$        $(\partial M_{NS} = -\varepsilon \partial M_2)$

Bhattacharya & van den Heuvel (1991)  
 Tauris (1996)  
 Soberman, Phinney & van den Heuvel (1997)  
 Tauris & van den Heuvel (2006)  
 Tauris & van den Heuvel (2022)

# Solution.....

Integration of the OAMB eq. for mass transfer/loss:

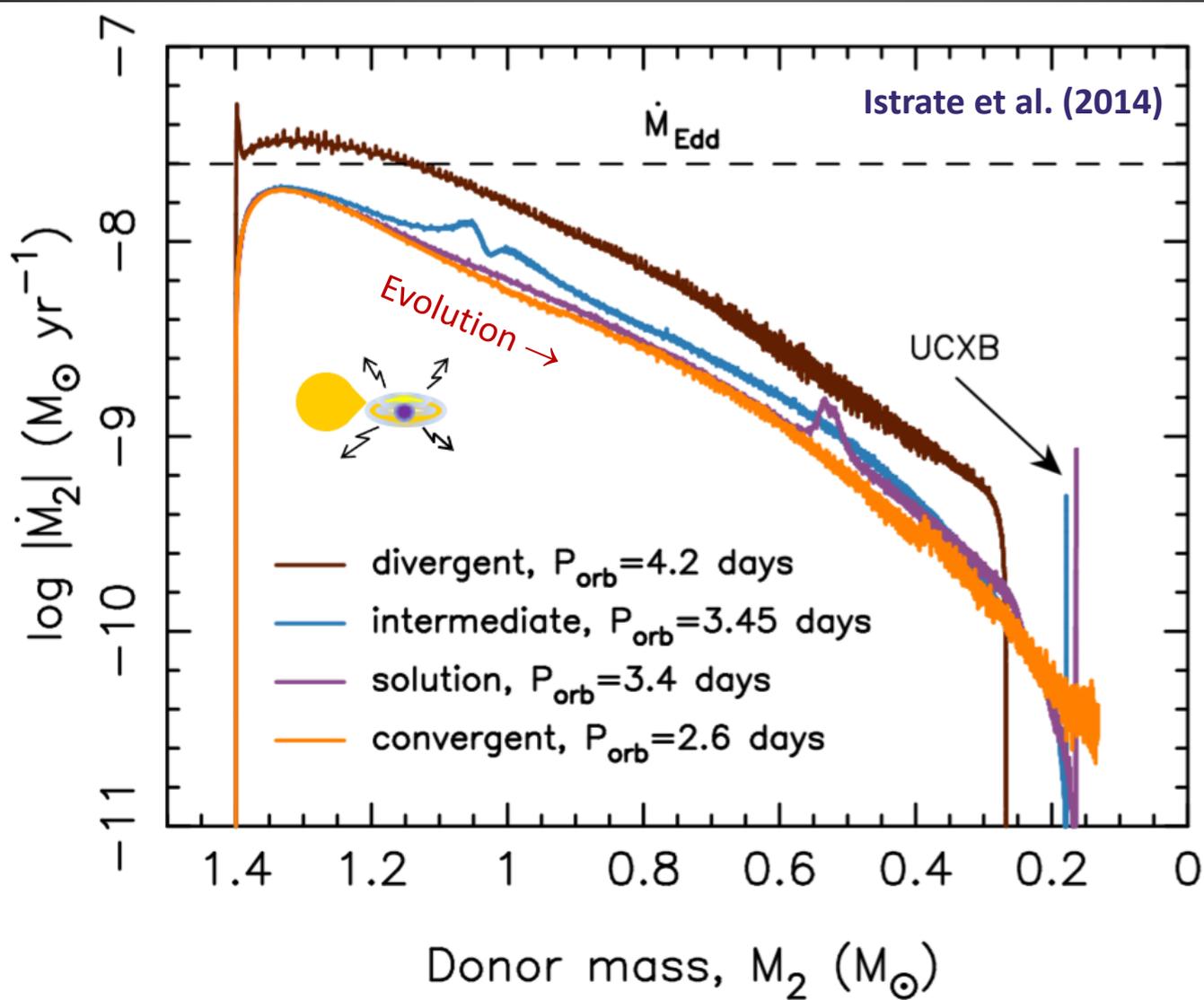
$$\frac{a}{a_0} = \Gamma_{ls} \left( \frac{q}{q_0} \right)^{2(\alpha+\gamma\delta-1)} \left( \frac{q+1}{q_0+1} \right)^{\frac{-\alpha-\beta+\delta}{1-\varepsilon}} \left( \frac{\varepsilon q+1}{\varepsilon q_0+1} \right)^{3+2\frac{\alpha\varepsilon^2+\beta+\gamma\delta(1-\varepsilon)^2}{\varepsilon(1-\varepsilon)}}$$

Tauris & van den Heuvel (2006)

mass ratio

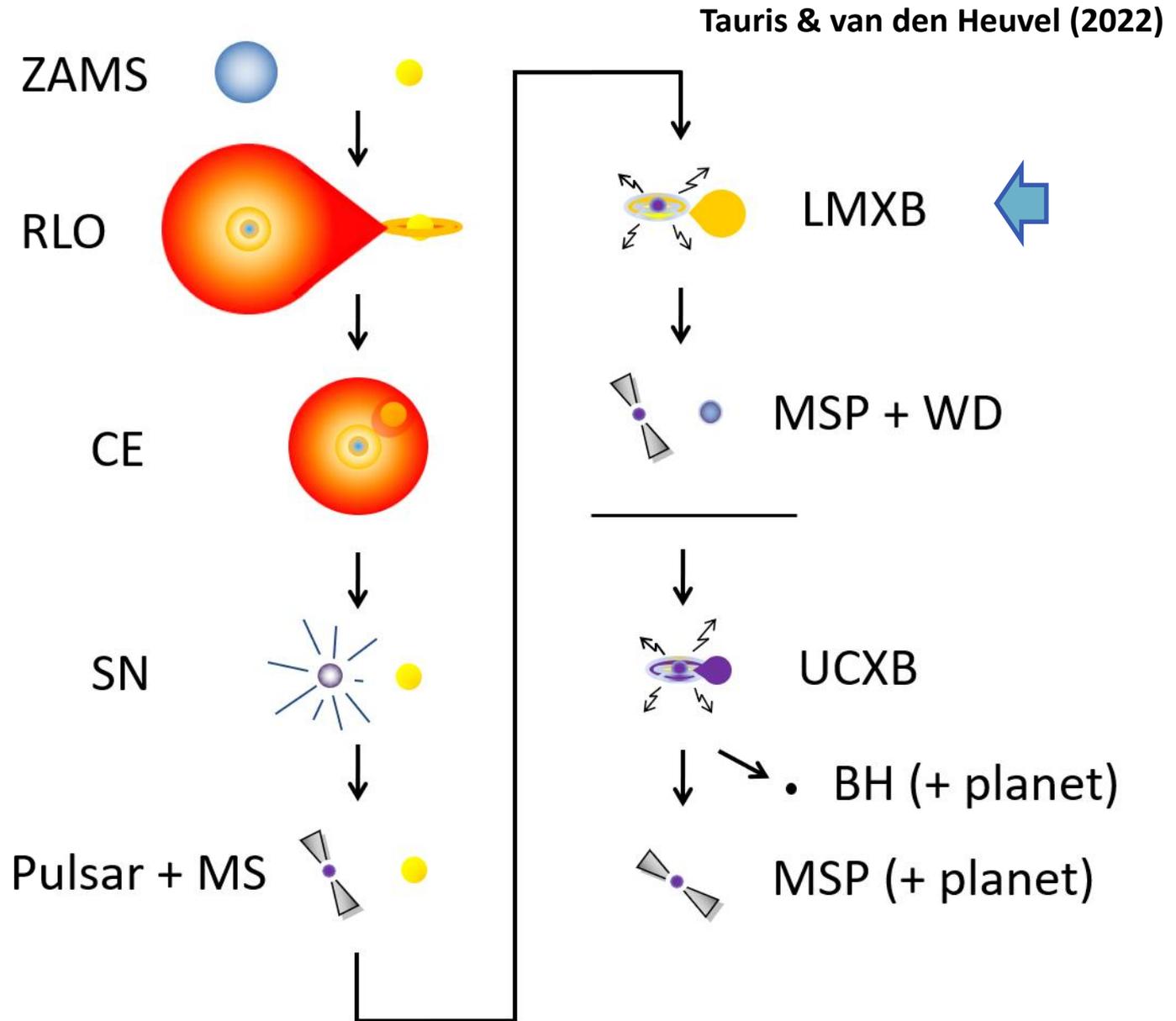
final separation (after RLO)

# Example of numerical LMXB calculation.....



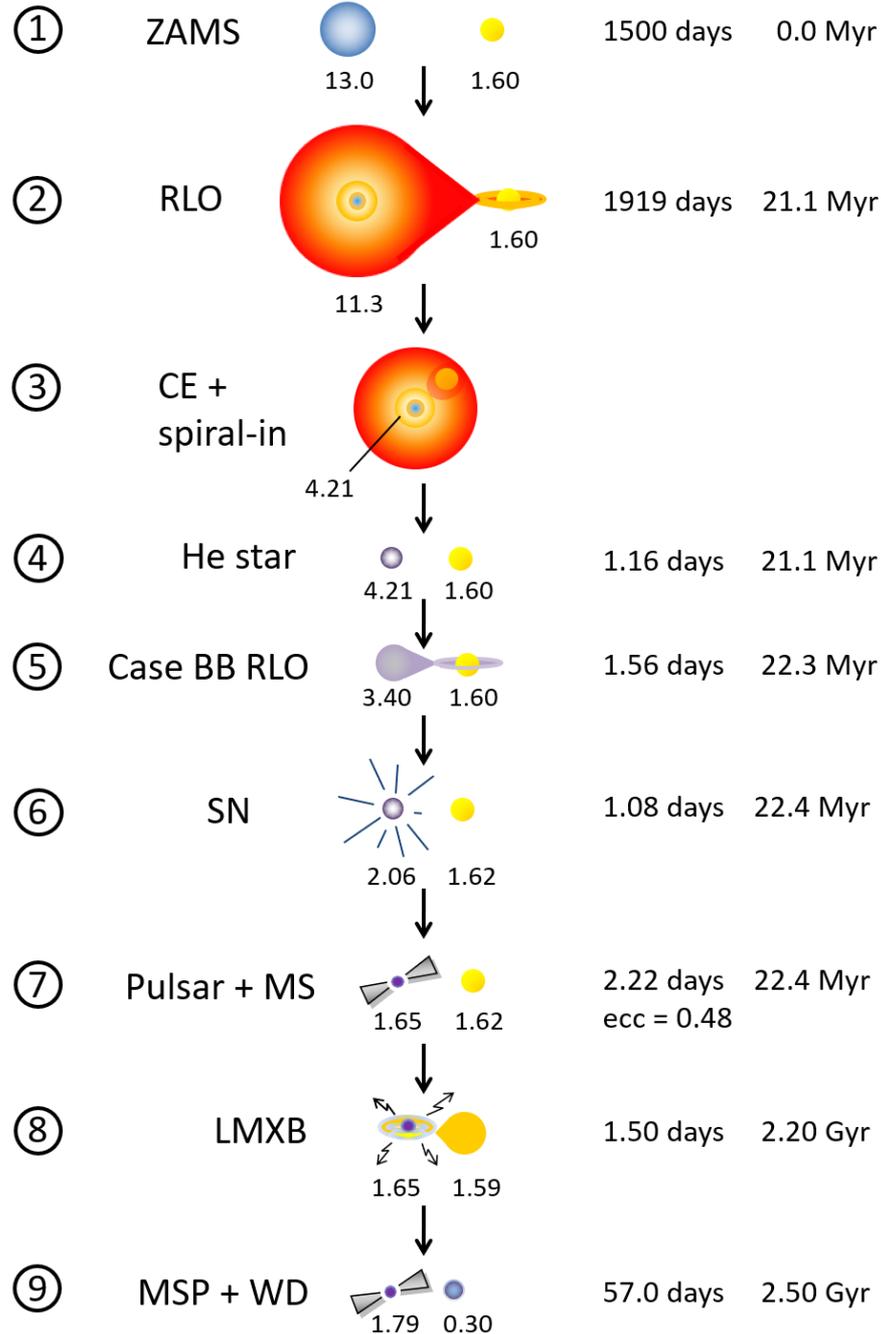
# LMXB

## Formation & Evolution



# LMXB

## Formation & Evolution in detail

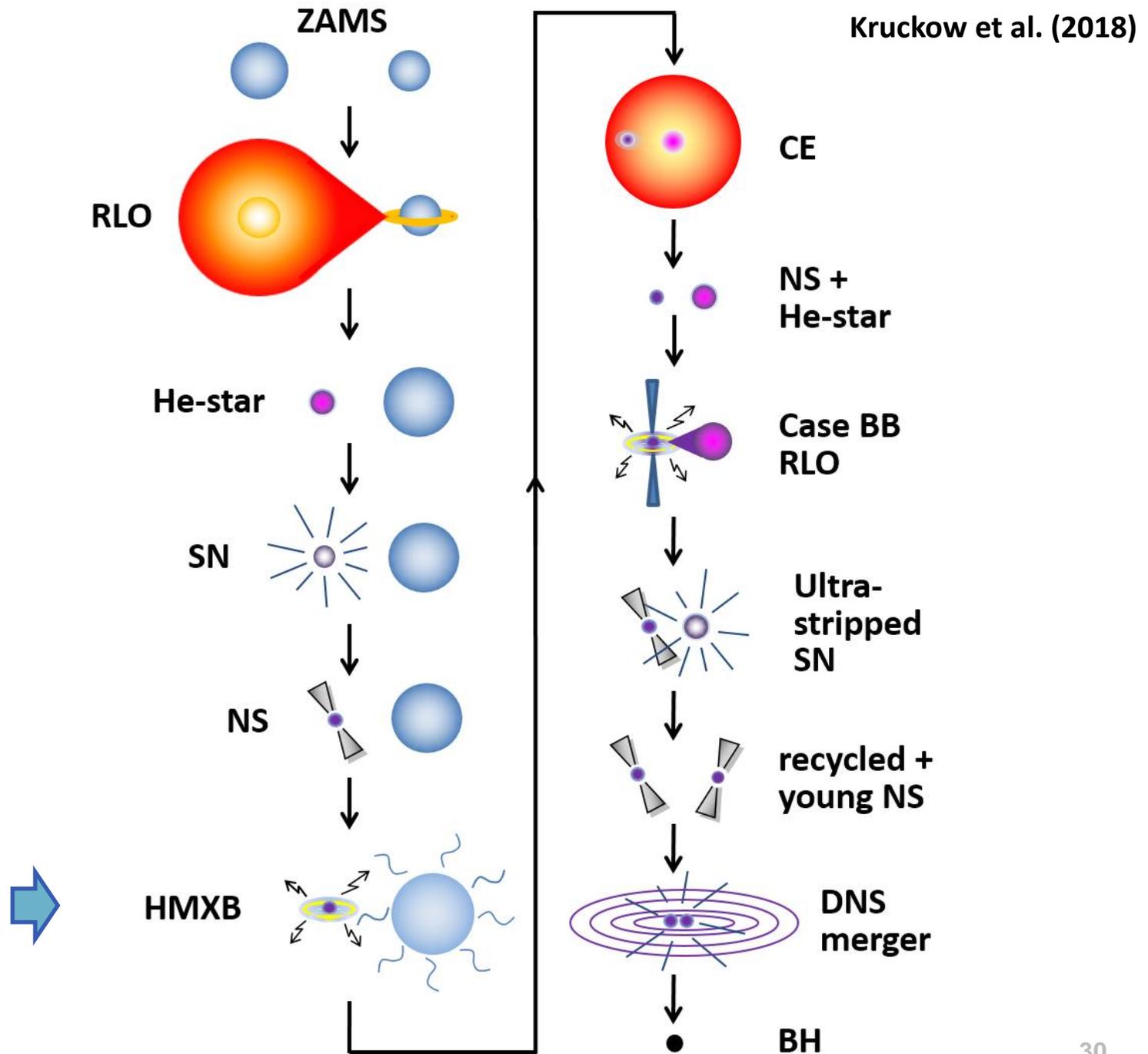


Tauris & van den Heuvel (2022)



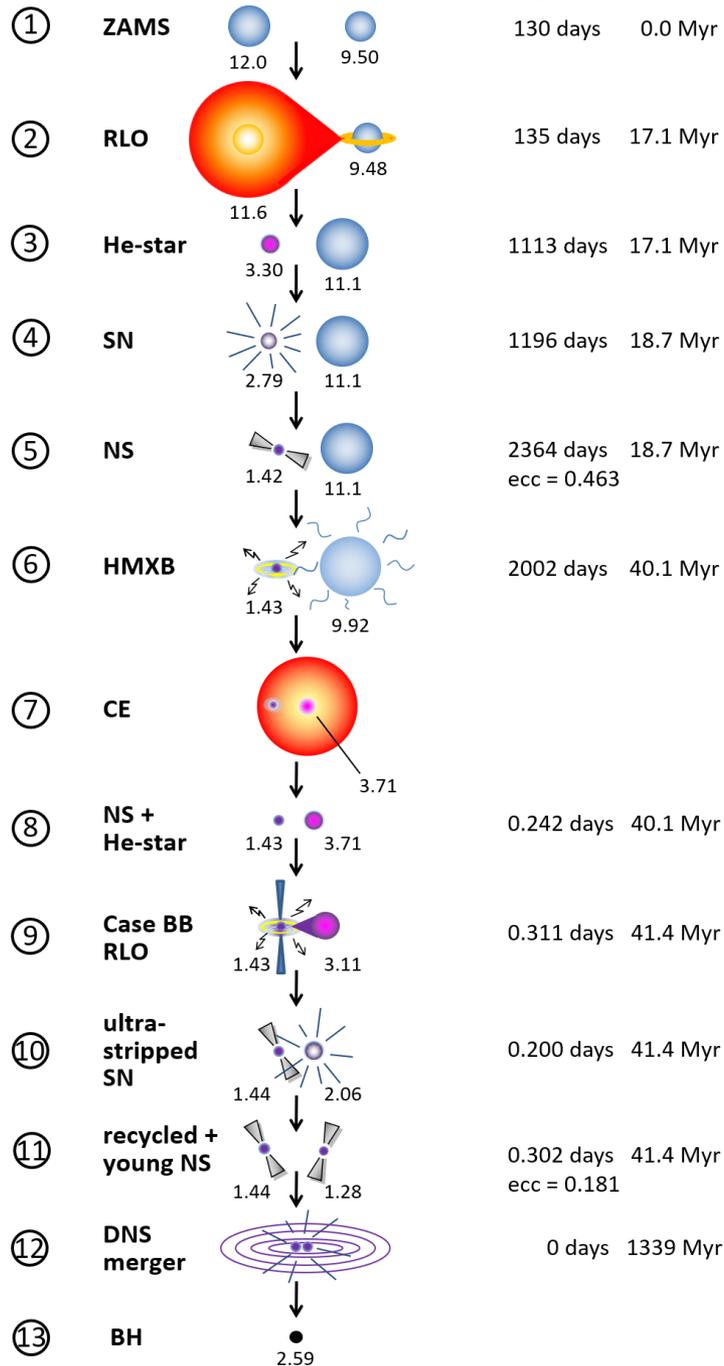
# HMXB

## Formation & Evolution



# HMXB

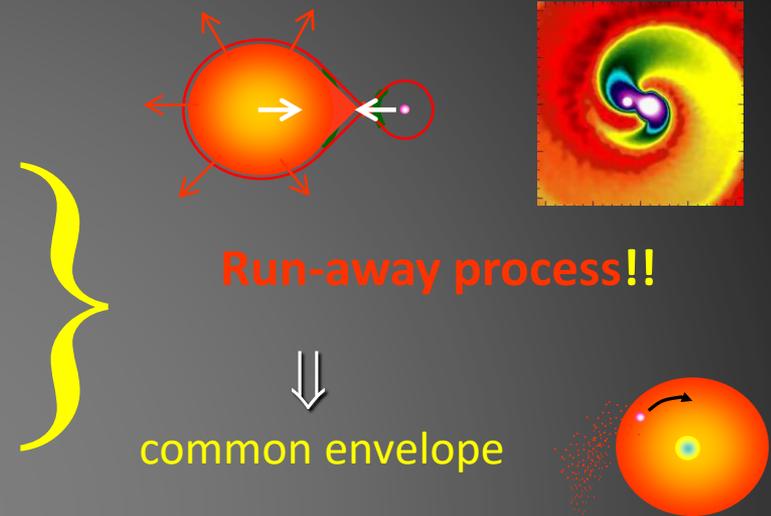
## Formation & Evolution in detail



# Common-Envelope + Spiral-in Evolution

Dynamically unstable mass transfer:

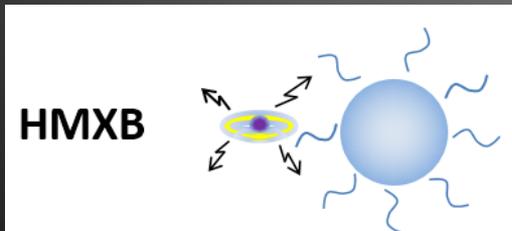
- deep convective envelope of donor star  
(rapid expansion in response to mass loss)
- $M_{\text{donor}} > M_{\text{accretor}}$   
(orbit shrinks in response to mass loss)



drag force  $\rightarrow$  dissipation of orb. ang. mom. + deposition of  $E_{\text{orb}}$  in the envelope

Outcome HMXBs  $\rightarrow$  CE:

- CE: huge reduction of orbital separation



rejection of stellar envelope  
(NS/BH orbiting a naked helium star)

merging of NS/BH + core  
(Thorne-Zytkow object / BH)

# COMMON-ENVELOPE EVOLUTION

$$\dot{E}_{orb} = - \frac{GM_{donor} M_{NS}}{2a^2} \frac{da}{dt} = \xi(\mu) \pi R_{acc}^2 \rho_{donor} v^3$$

Dissipation of  $E_{orb}$  by drag force (Bondi & Hoyle 1944)

$$E_{env} \equiv \alpha_{CE} \Delta E_{orb}$$

Webbink (1984)

$$\alpha_{CE} = 0.3 \sim 1$$

efficiency parameter



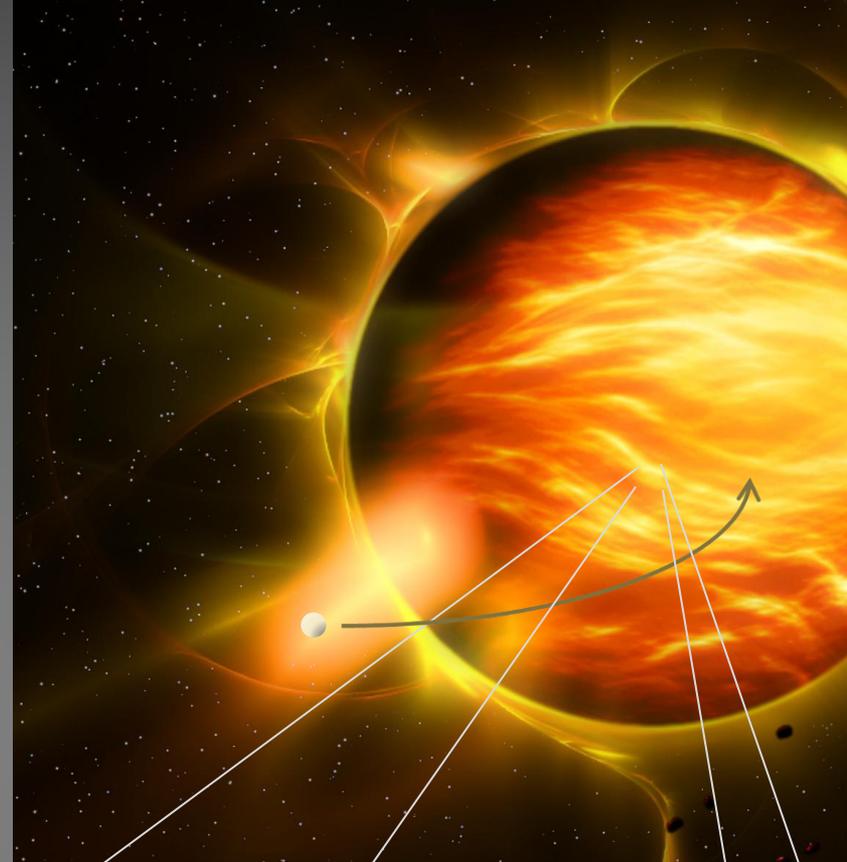
$$E_{env} = - \int_{M_{core}}^{M_{donor}} \frac{GM(r)}{r} dm + \alpha_{th} \int_{M_{core}}^{M_{donor}} U dm$$

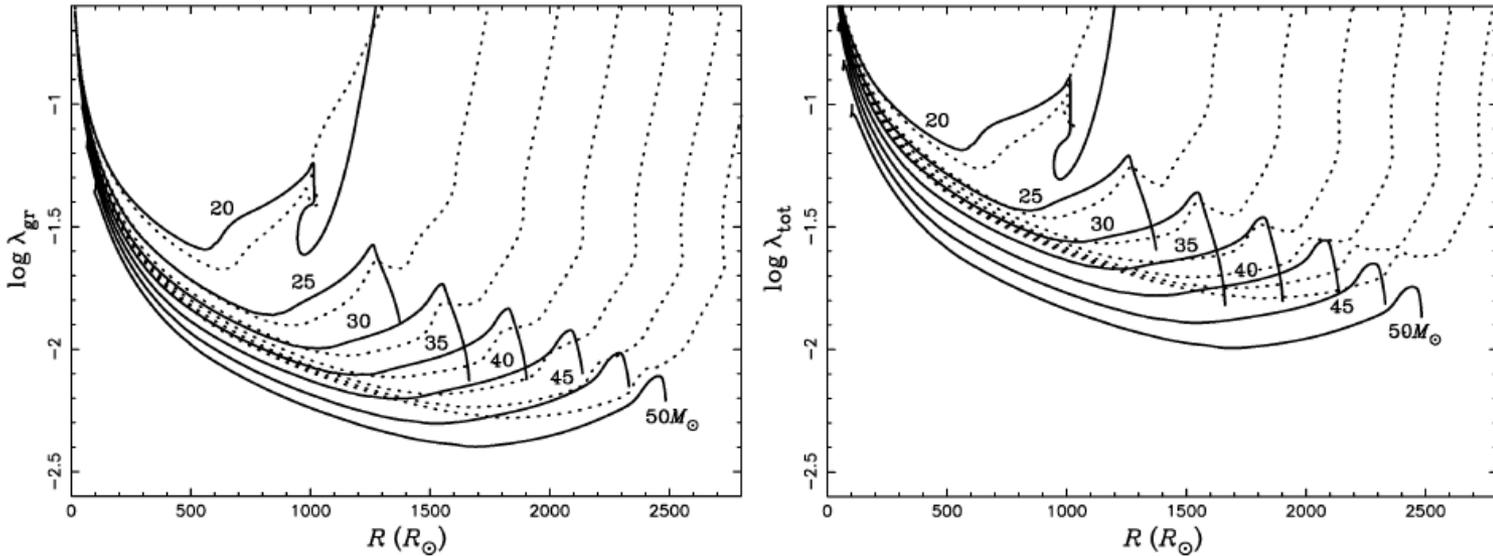
gravitational binding energy

internal thermodynamic energy

Han et al. (1994, 1995)

- thermal energy
- energy of radiation
- ionization energy
- Fermi energy of  $e^-$ -gas





**Figure 1.** The envelope structure parameter  $\lambda$  as a function of stellar radius for different masses as indicated after hydrogen has been exhausted in the core. In the panels on the left,  $\lambda$  only includes the gravitational binding energy, while on the right  $\lambda$  includes both the gravitational binding energy and the energy of the stellar wind (similar to Dewi & Tauris (2000)). The dotted curves are calculated without inclusion of a stellar wind. Note that in this case the models reach the largest radii attained by the models.

$$E_{env} \equiv - \frac{GM_{donor} M_{env}}{\lambda R_L}$$

de Kool (1990)

$$\lambda = 0.01 \sim 100$$

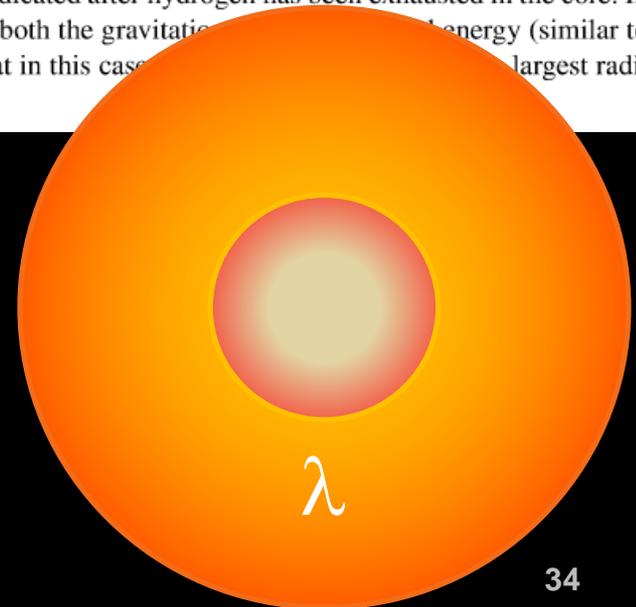
Dewi & Tauris (2000, 2001)

Podsiadlowski et al. (2003)

Xu & Li (2010)

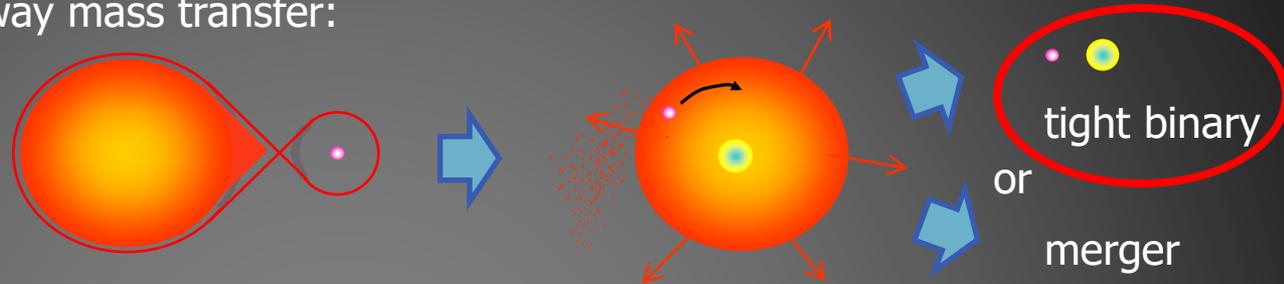
Loveridge et al. (2011)

Kruckow et al. (2016)



# Common-Envelope + Spiral-in Evolution

Dynamically unstable run-away mass transfer:



$$E_{\text{env}} \equiv \alpha_{\text{CE}} \Delta E_{\text{orb}}$$

$$\alpha_{\text{CE}} = 0.3 \sim 1$$

$$E_{\text{env}} \equiv -\frac{GM_{\text{donor}} M_{\text{env}}}{\lambda R_L}$$

$$\Delta E_{\text{orb}} = -\frac{GM_{\text{core}} M_{\text{NS}}}{2a_f} + \frac{GM_{\text{donor}} M_{\text{NS}}}{2a_i}$$

$$\lambda = 0.01 \sim 100$$

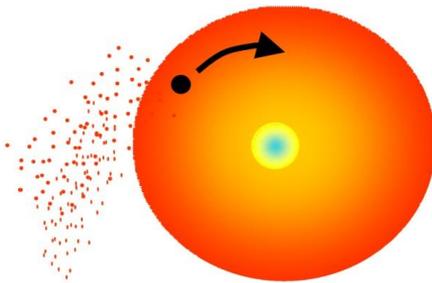
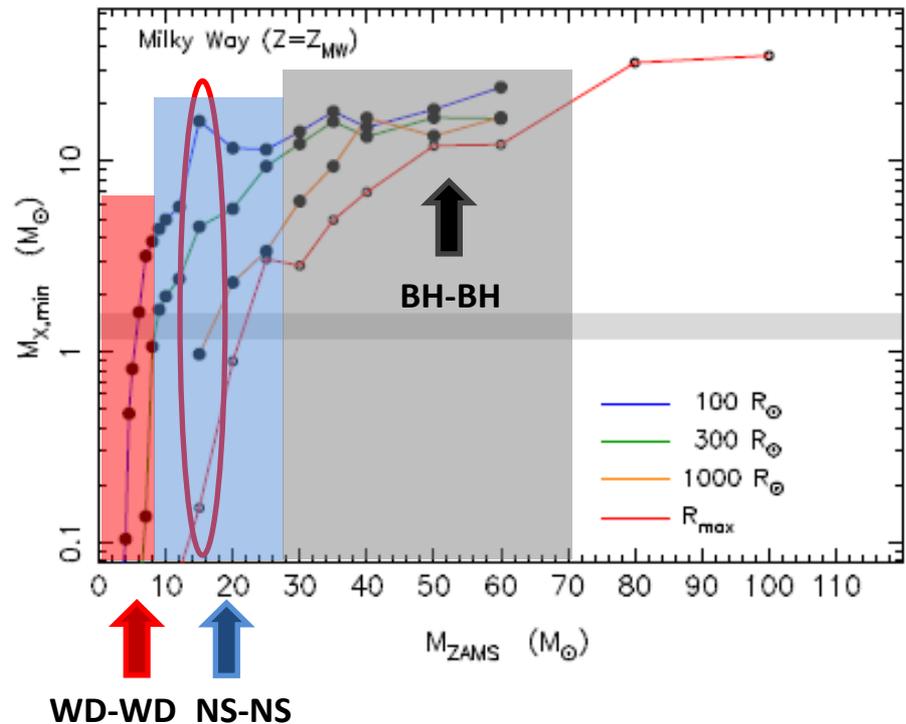
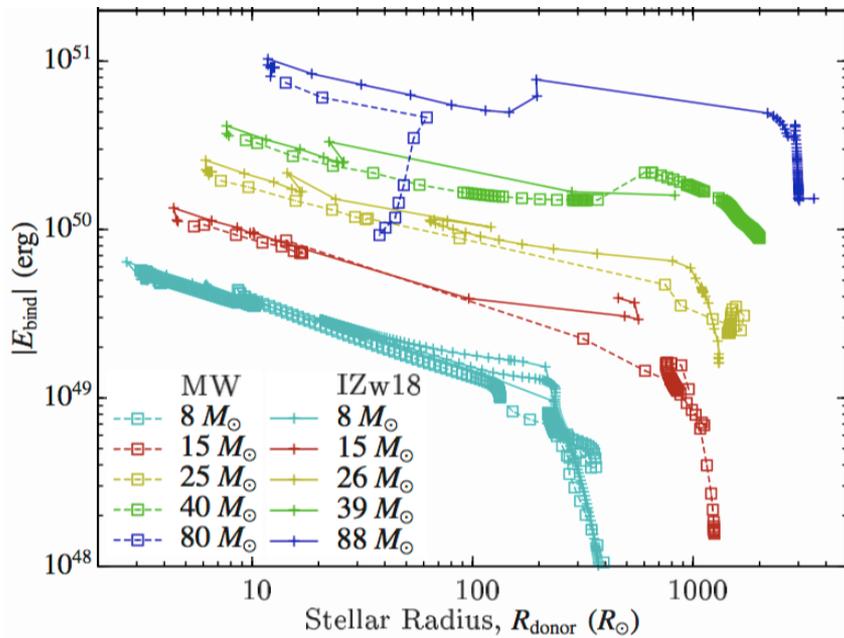
$$\frac{a_f}{a_i} = \frac{M_{\text{core}} M_2}{M_{\text{donor}} M_2 + 2M_{\text{env}} / (\eta_{\text{CE}} \lambda r_L)} \Rightarrow \frac{a_f}{a_i} \approx 10^{-3} - 10^{-2}$$

# Can an in-spiralling BH or NS eject the envelope of a massive star?

Minimum mass of in-spiralling star to successfully eject the envelope?



Kruckow, Tauris, Langer, Szecsi, Marchant & Podsiadlowski (2016), A&A  
*Common-envelope ejection in massive binary stars*  
 – Implications for the progenitors of GW150914 and GW151226

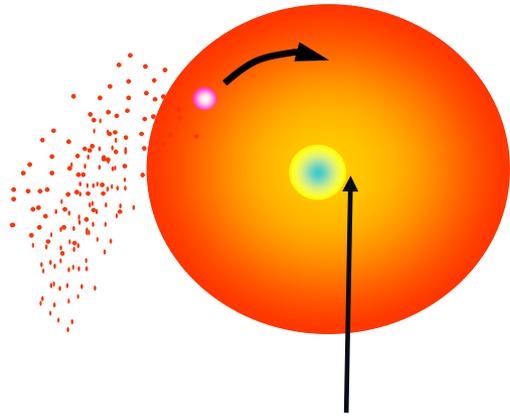


Example:

- $M = 15 M_{\odot} : R < 500 R_{\odot}$  no CE ejection  $\rightarrow$  coalescence
- $M = 15 M_{\odot} : R > 500 R_{\odot}$  successful CE ejection

# CE EJECTION

Bifurcation point ?



Bifurcation point ?

Tauris & Dewi (2001)

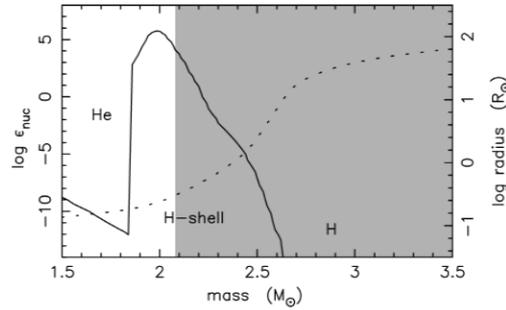
Ivanova (2011)

Kruckow et al. (2016)

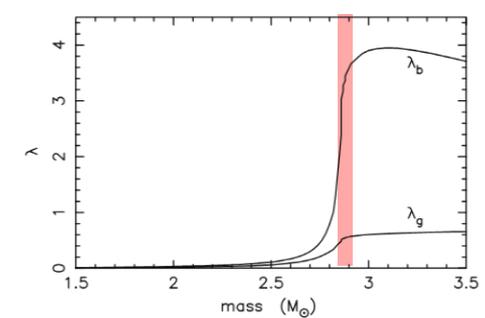
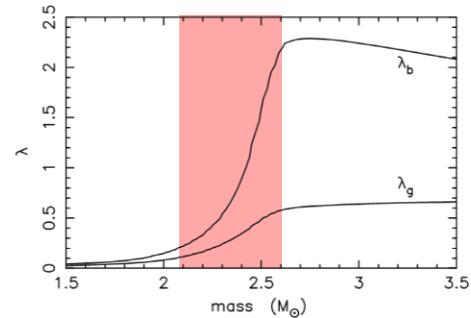
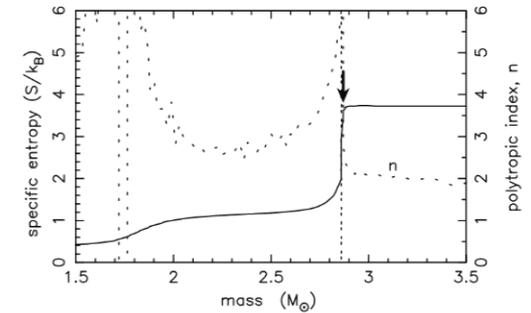
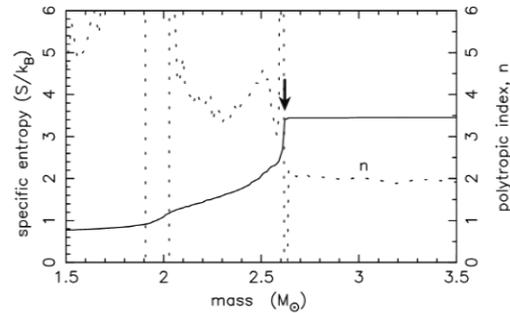
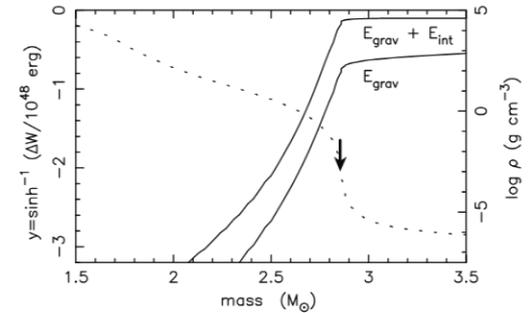
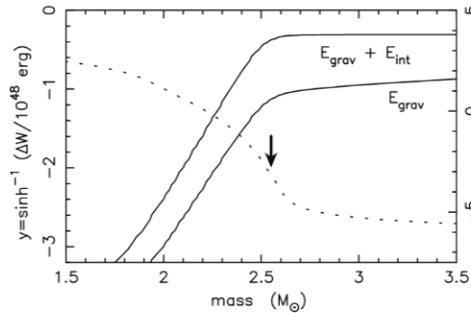
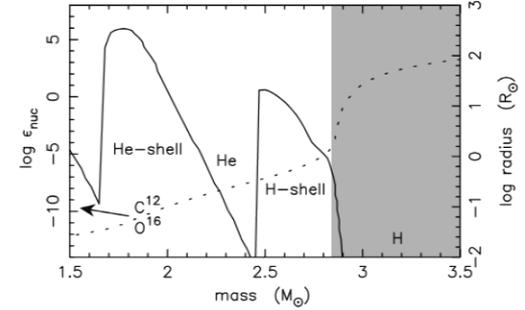
Marchant et al. (2021)

Tauris & Dewi (2001)

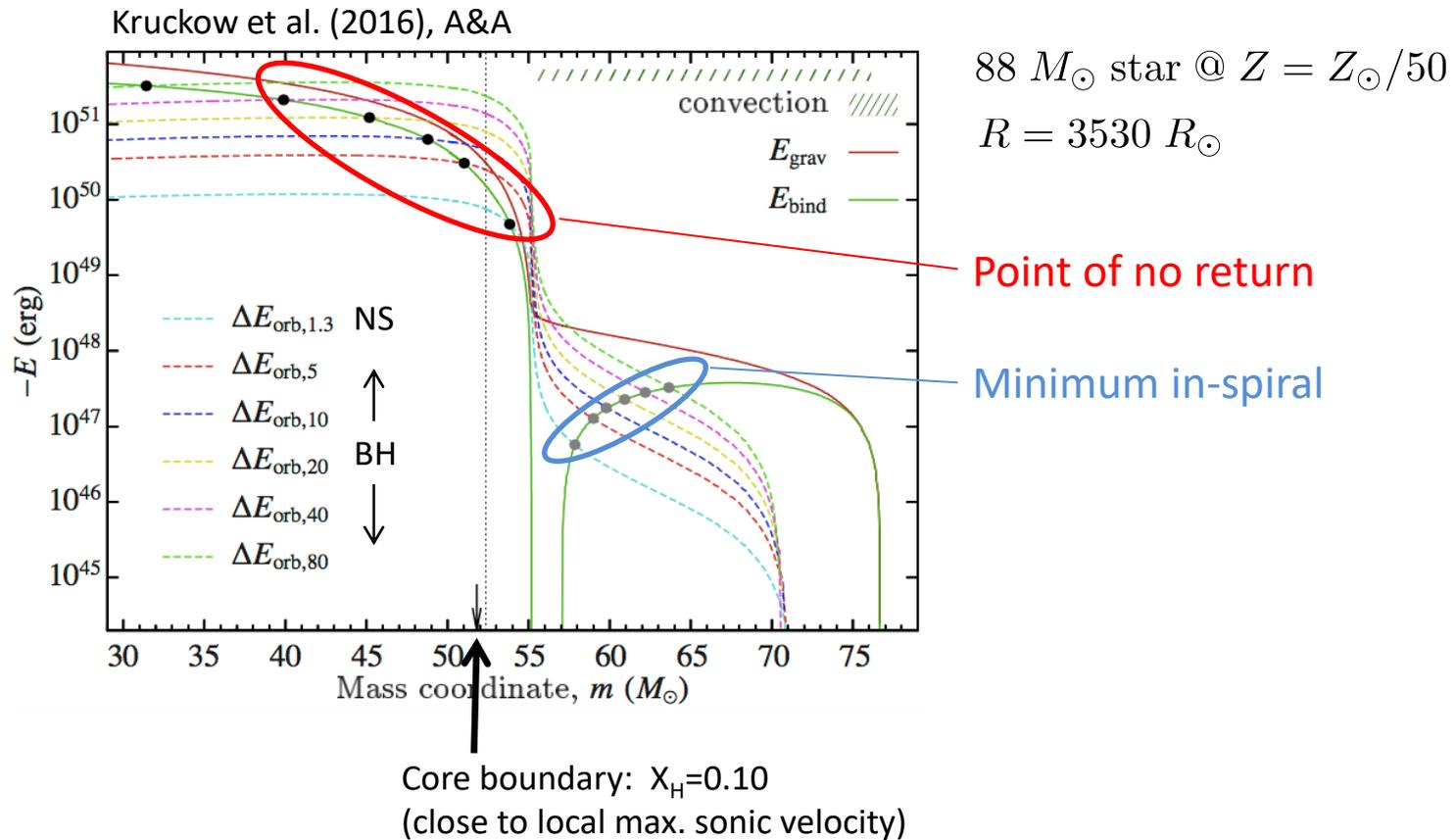
10  $M_{\odot}$  Tip of RGB ( $R=374 R_{\odot}$ )

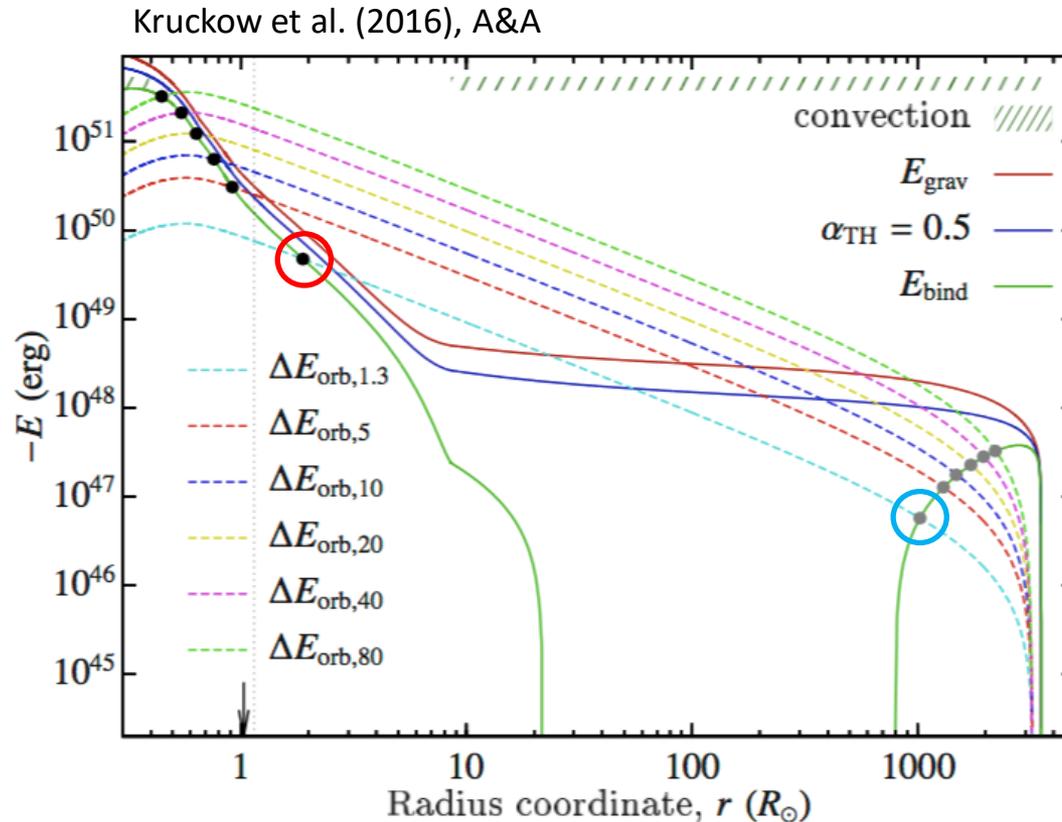


10  $M_{\odot}$  Tip of AGB ( $R=588 R_{\odot}$ )



## Where does the envelope ejection terminate?

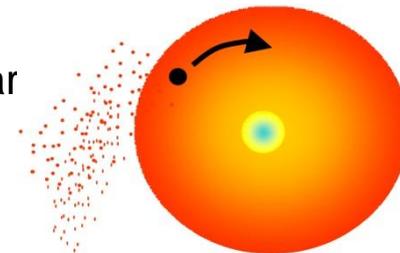




Difference in mass coordinate of about  $4 M_{\odot}$   
 corresponds to a **radius difference** by a **factor 500!**  
 Extremely important for the final orbital separation.

Additional **energy sources** to help envelope ejection:

- **Recombination energy** from outer (cold) layers of donor star (from H, He, H<sub>2</sub>)
- Released **accretion energy** from BH during in-spiral



$$\Delta E_{\text{acc}} = \eta \dot{M}_{\text{Edd}} c^2 \tau_{\text{CE}}$$

$$\dot{M}_{\text{Edd}} = 4.4 \times 10^{-9} M_{\odot} \text{ yr}^{-1} \left( \frac{M_{\text{BH}}}{M_{\odot}} \right) \frac{r_*}{(1 + X_H)}$$

$$\Delta E_{\text{acc}} = 1.6 \times 10^{48} \text{ erg} \left( \frac{M_{\text{BH}}}{M_{\odot}} \right) \left( \frac{\tau_{\text{CE}}}{1000 \text{ yr}} \right) \left( \frac{\eta}{0.20} \right) \frac{r_*}{(1 + X_H)}$$

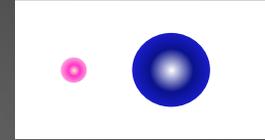
$$r_* = R_{\text{ISCO}} / (GM_{\text{BH}}/c^2)$$

Kruckow et al. (2016)

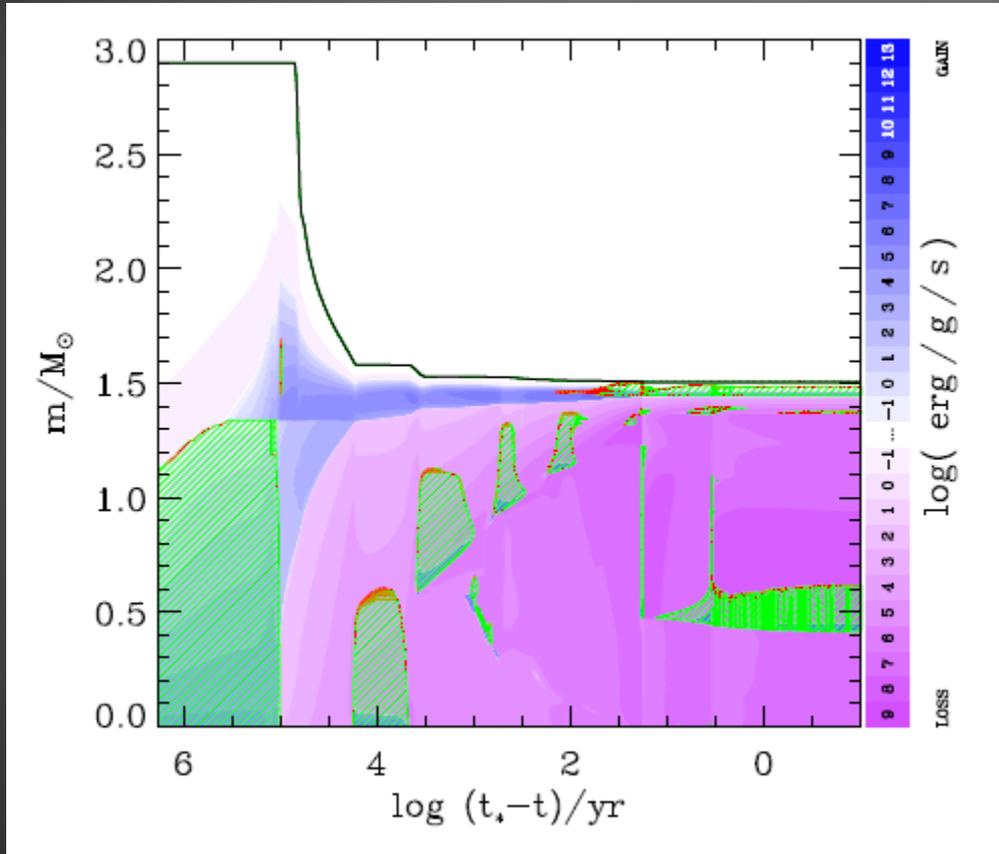
More to learn from ongoing and future hydrodynamical simulations

# Ultra-stripped pre-SN metal core

post-CE  
Case BB RLO



Tauris et al. (2013)



Post-CE evolution of  
He-star + NS in close orbit  
⇒ bare, pre-SN core just  
above Chandrasekhar mass

⇒ Iron core-collapse SN  
with very little ejecta  
+ formation of low-mass NSs  
in NS+NS system

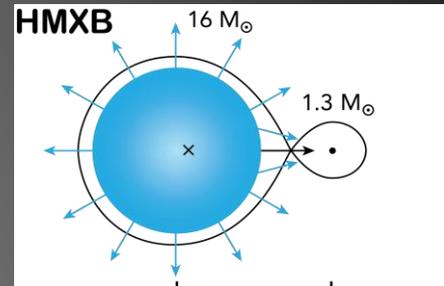
# Intermediate-Mass X-ray Binaries

## Why are so few IMXBs observed ?

HMXB: wind accretion (beginning atmospheric RLO)

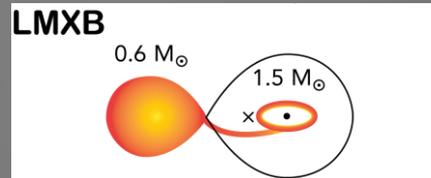
RLO is dynamically unstable and a CE forms

$$M_2 > 10 M_{\odot}$$



LMXB: stable RLO

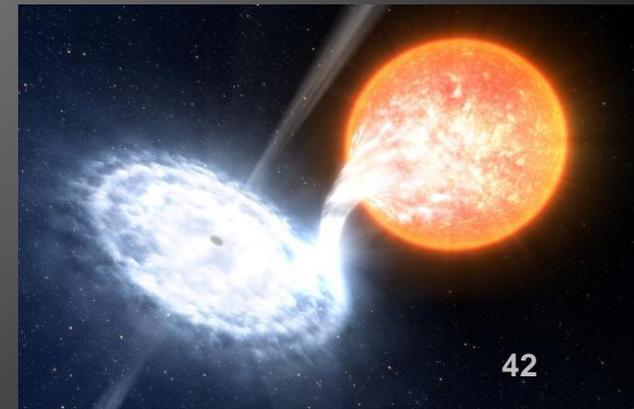
$$M_2 \leq 1.5 M_{\odot}$$



IMXB: wind accretion is too weak, and

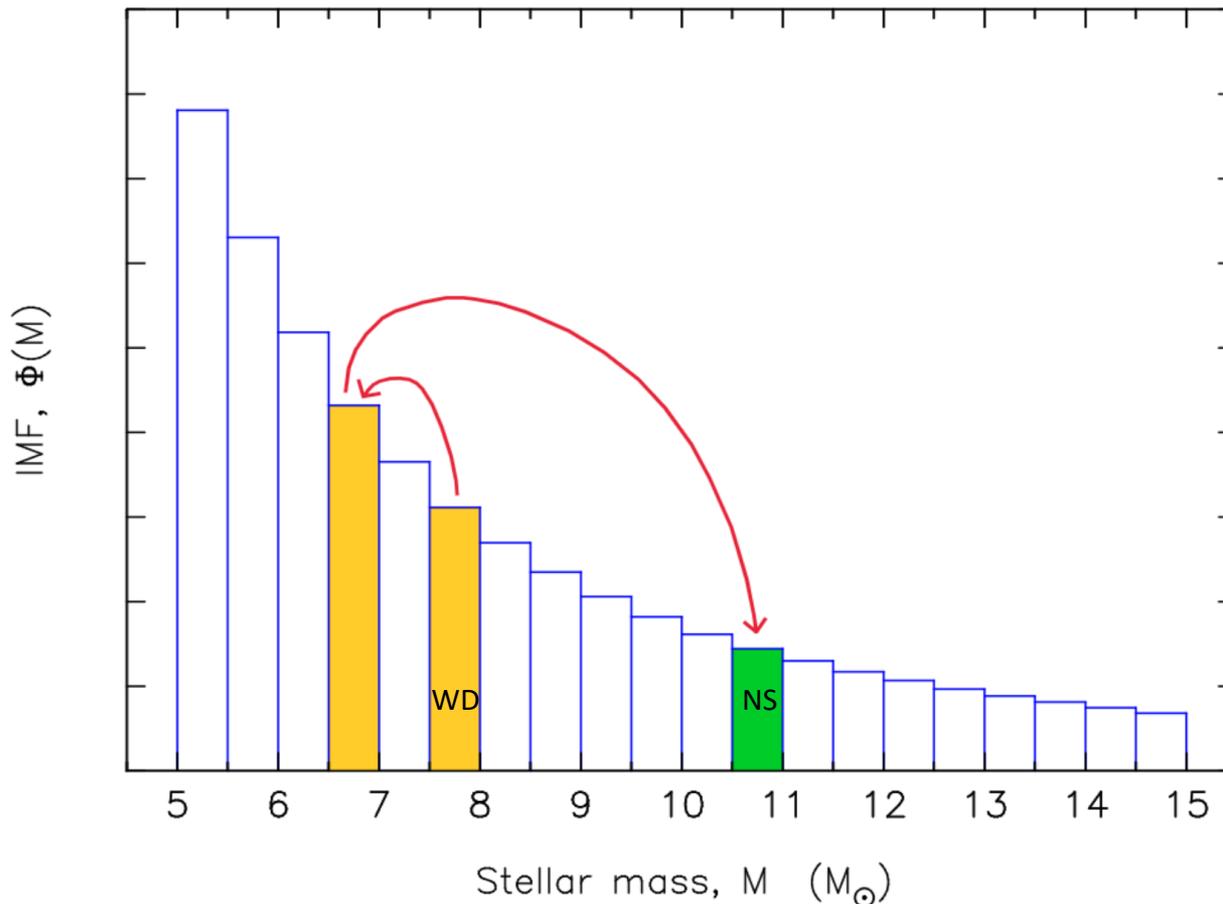
RLO is often unstable (or very short)

**But evidence of post-IMXBs  
from pulsar+CO WD binaries**



# Binaries undergoing mass reversal

Tauris & van den Heuvel (2021)



producing (WD-NS) systems

first formed

second formed  
(non-recycled, young,  
eccentric orbit from SN)

# LECTURE 2

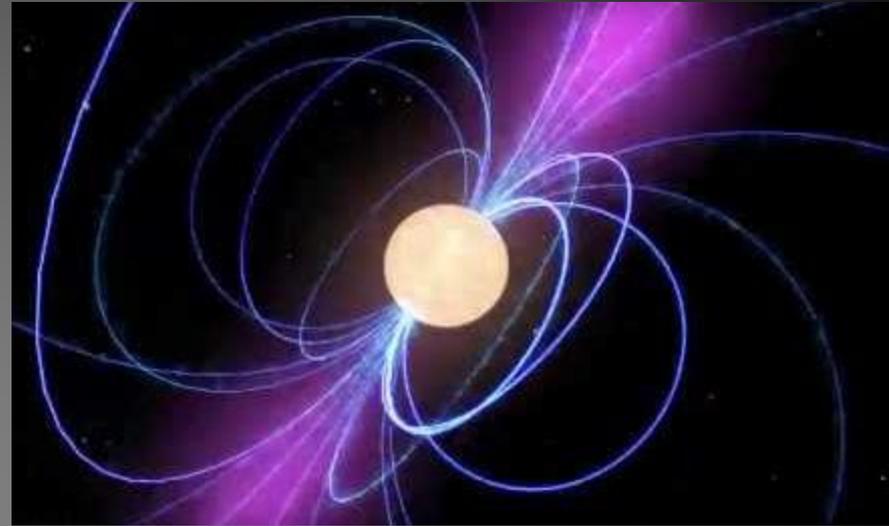
## Recycling and Millisecond Pulsars

### MSPs

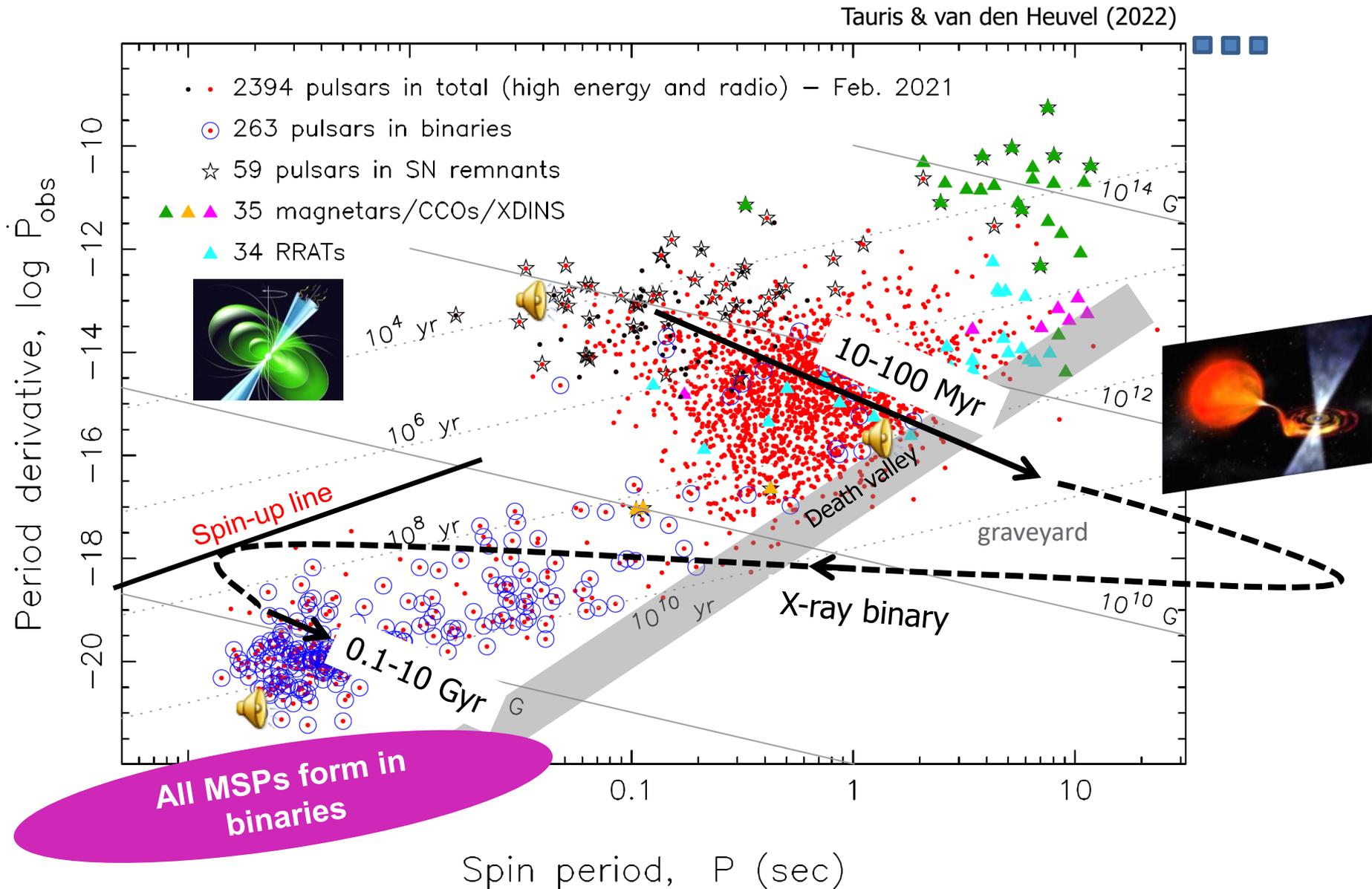
- How fast can they spin?
- Why are their B-field strengths weak?
- How much mass do they need to accrete?
- What are their orbital periods?
- How old are they?
- Where are they located (+ kinematics)?
- What is the nature of their companion stars?



How do MSPs form?



# Recycling pulsars - A detour in the P-Pdot diagram



# Millisecond pulsars - a binary formation scenario

- Rapid spin:  $P < 30 \text{ ms}$
- Small period derivative:  $\dot{P} < 10^{-17} \text{ s s}^{-1}$

$$\dot{J} = \frac{d}{dt} |\vec{r} \times \vec{p}|$$

Solution:

- Accretion of mass

$$N = \dot{J}_* \equiv \frac{d}{dt} (I \Omega_*) = \dot{M}_* \sqrt{GM_* r_A} \xi$$

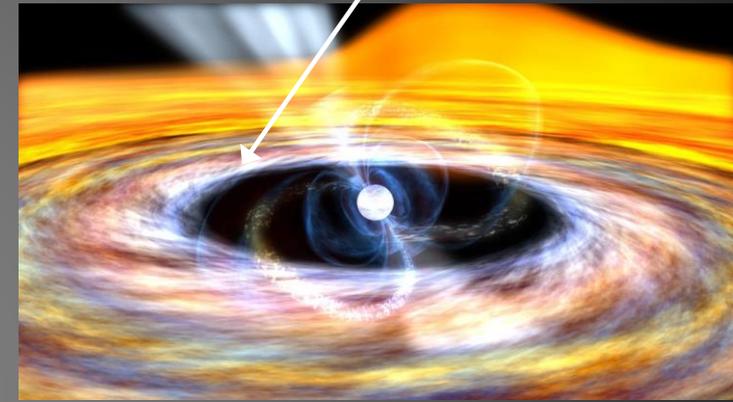
Lamb, Pethick & Pines (1973)  
Ghosh & Lamb (1979, 1992)

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \frac{c^2}{4\pi} \nabla \times \left( \frac{1}{\sigma} \times \nabla \times \vec{B} \right)$$

Geppert & Urpin (1994); Konar & Bhattacharya (1997)

$$B = \sqrt{\frac{3c^3 I_{NS}}{8\pi^2 R_{NS}^6} P \dot{P}}$$

Magnetic-dipole model



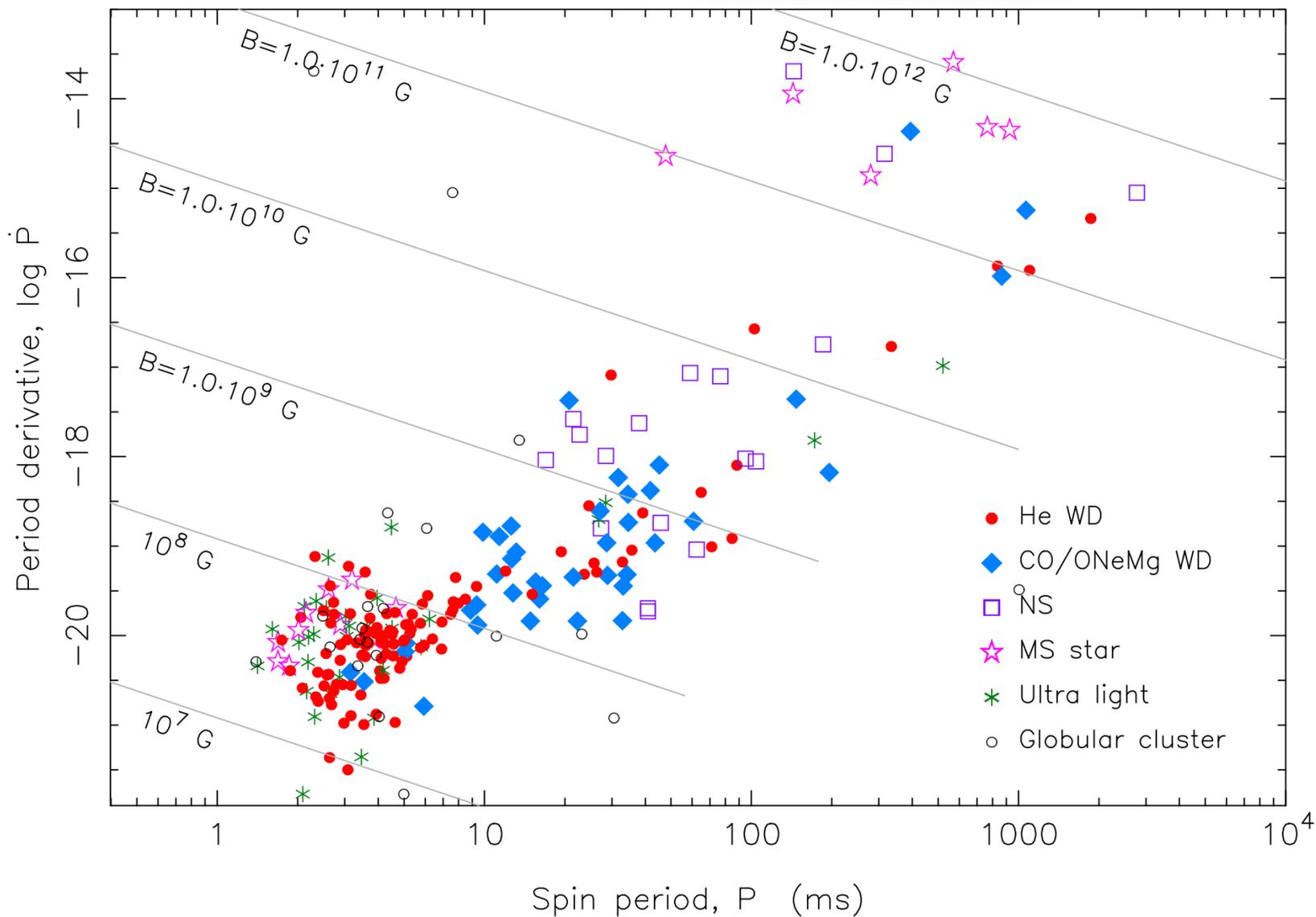
**Accretion induced B-field decay**

- Ohmic dissipation/diffusion
- Flux tube expulsion via spin-down
- B-field burial (screening)

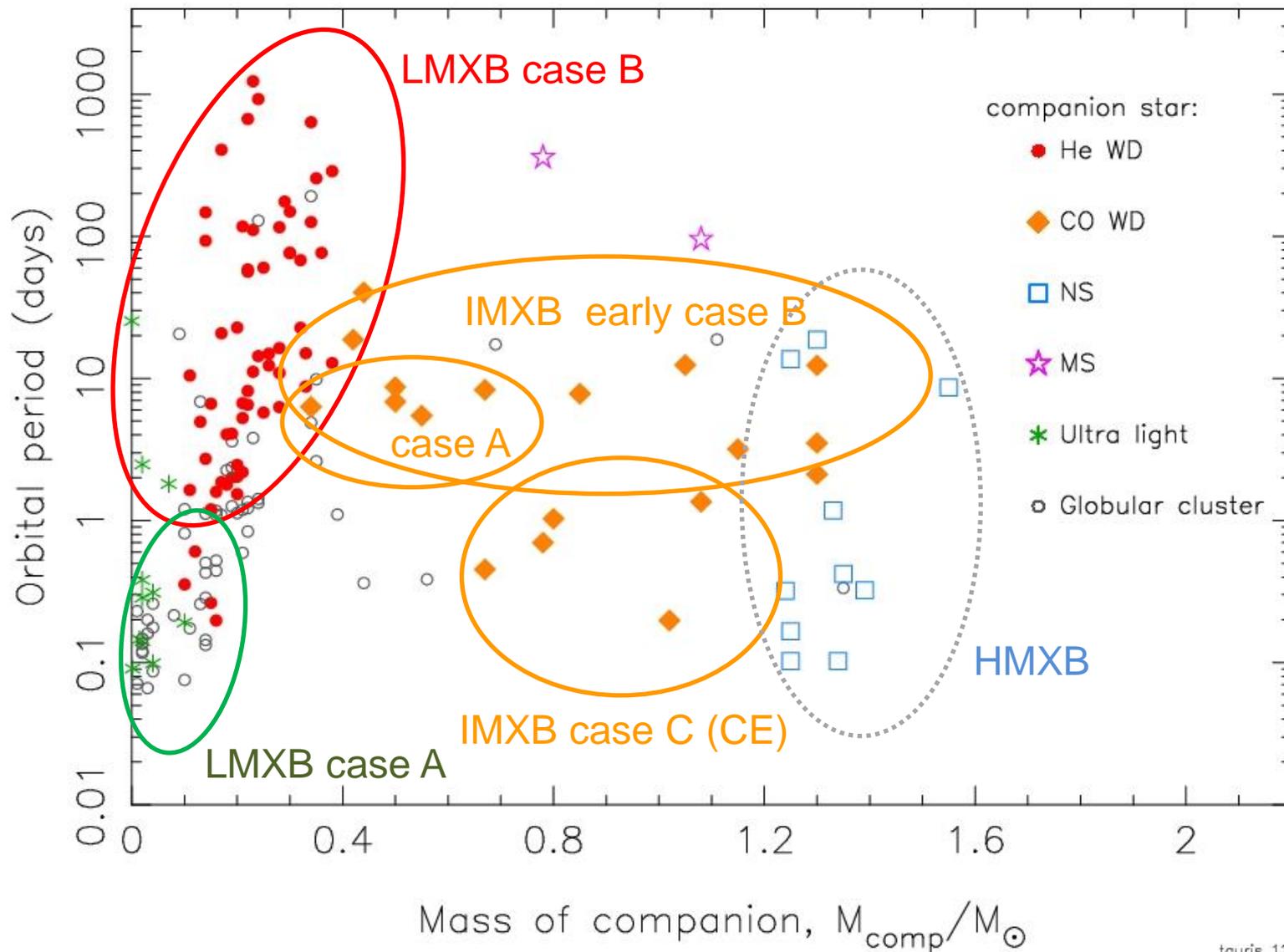
# Pulsar companion stars

245 binary pulsars with measured  $\dot{P}$

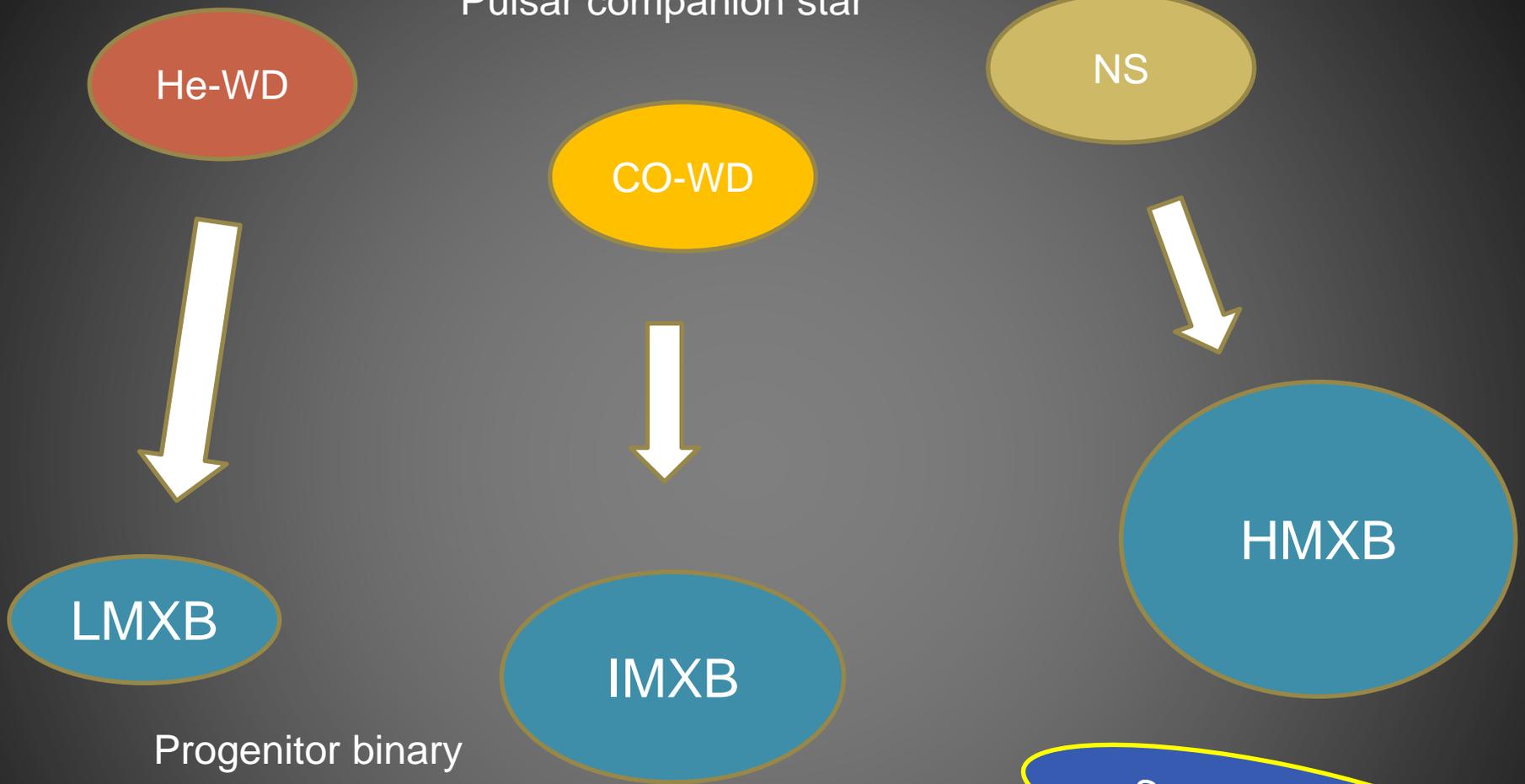
Tauris & van den Heuvel (2022)



# Origin of binary radio pulsars - Which X-ray Binaries?



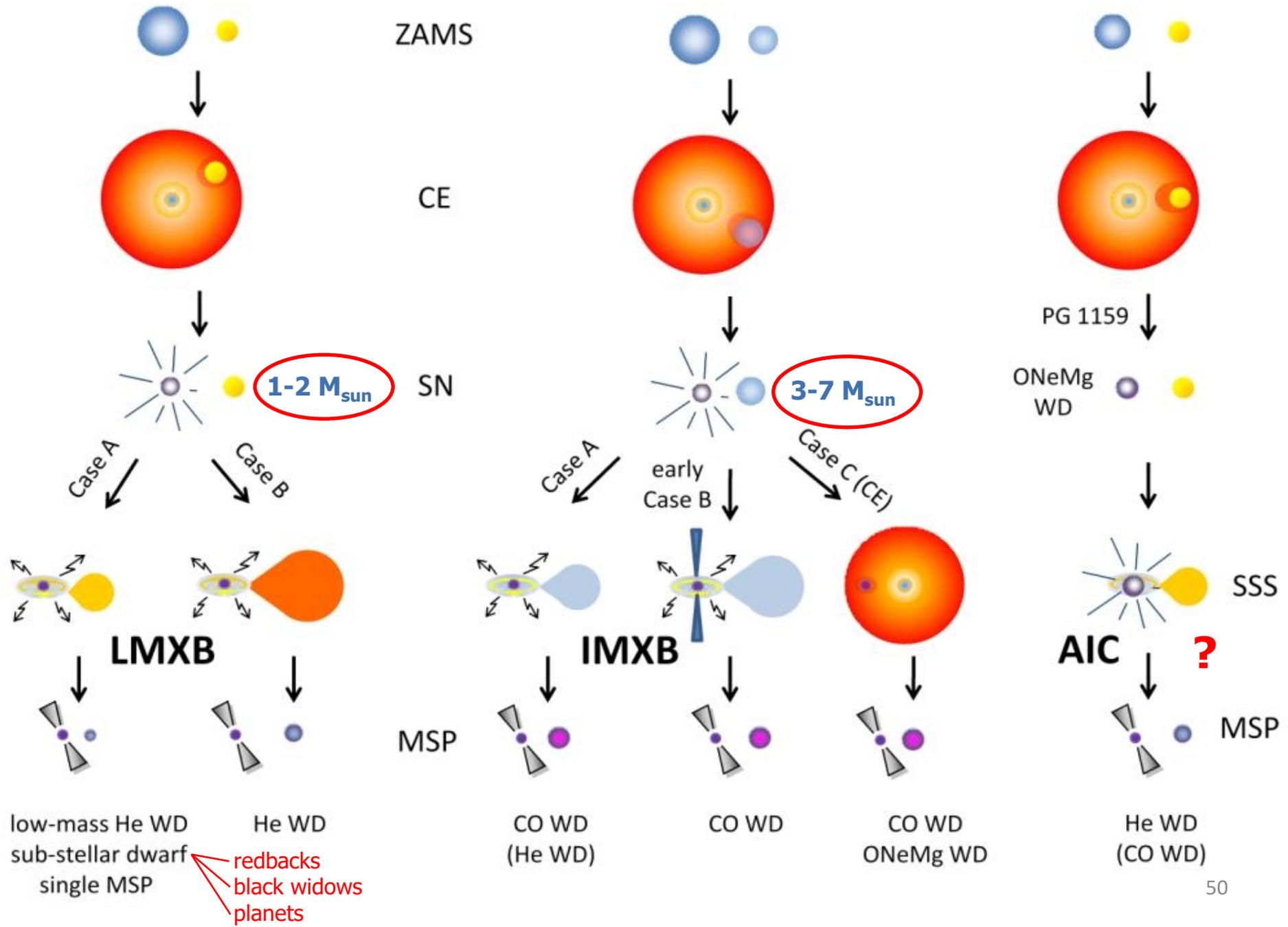
Pulsar companion star



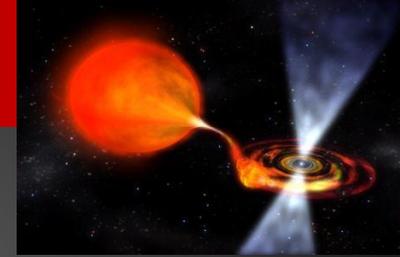
Progenitor binary

Stellar evolution

Initial orbital period is very important!



# LMXB bifurcation period



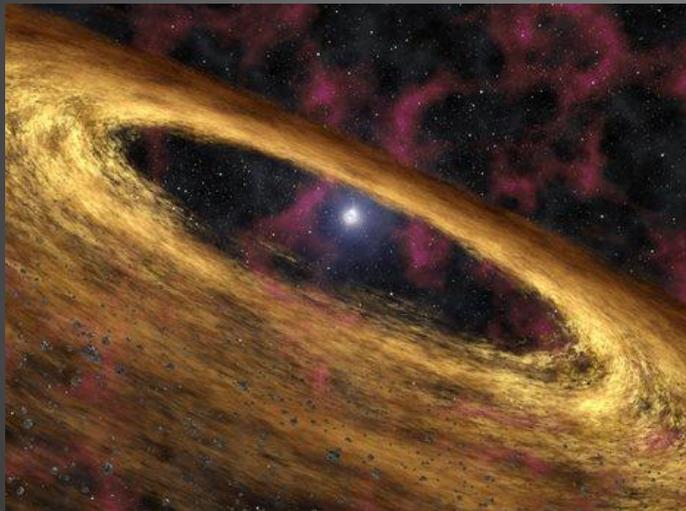
$$P_{orb} < P_{bif}$$

Converging

→ LMXB shorten their orbital period

Donor star still on main sequence

RLO driven by loss of  $J_{orb}$  (MB, GWs)



“Black widow” millisecond pulsars:

$$P_{orb} < 10 \text{ hrs} \quad M_{comp.} < 0.1 M_{\odot}$$

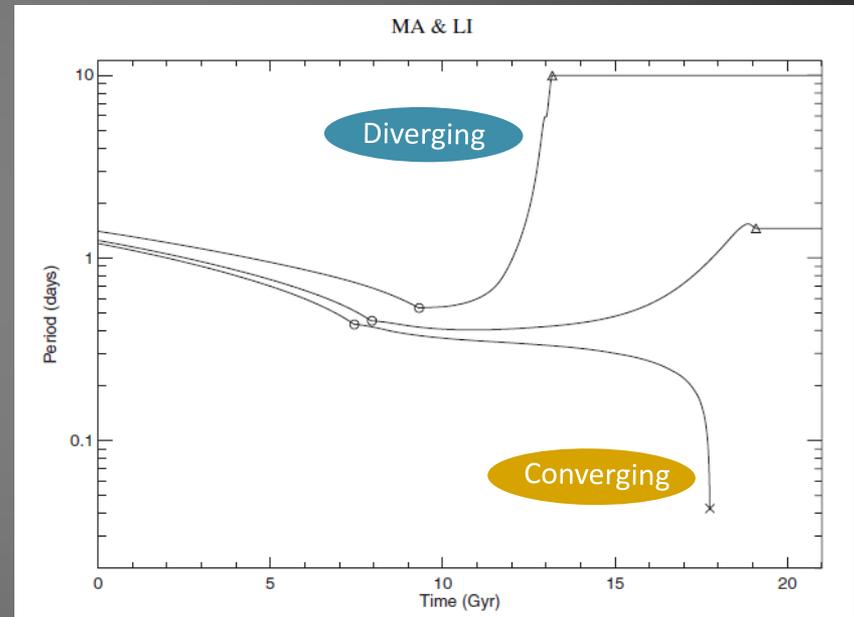
EoS?

Evaporation → single millisecond pulsars

?

- Tutukov et al. (1985)
- Pylser & Savonije (1988, 1989)
- Ma & Li (2009)
- Istrate et al. (2014)
- Chen et al. (2021)

$$P_{bif} \approx 1 \text{ day}$$



Roberts (2012)

Breton et al. (2013)

Chen et al. (2013)

# Single Millisecond Pulsars

All single millisecond pulsars are born in a binary system.

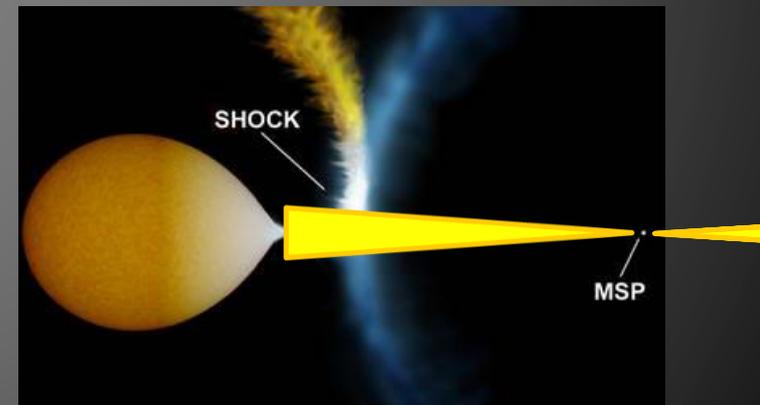
Once a recycled millisecond pulsar turns on its emission of ultra-relativistic particles, it is often able to completely evaporate its companion and thus end up as an isolated millisecond pulsar.

## Observational evidence:

- ✓ eclipsing MSPs with  $0.02 M_{\text{sun}}$  companions
- ✓ the "planetary pulsar", PSR 1257+12

$$\frac{1}{2} \dot{M}_2 v_{\text{esc}}^2 = f \dot{E}_{\text{psr}} \left( \frac{\pi R_2^2}{4\pi a^2} \right) \quad \tau \sim \frac{M_2}{\dot{M}_2}$$

evaporation timescale



# LMXB diverging systems

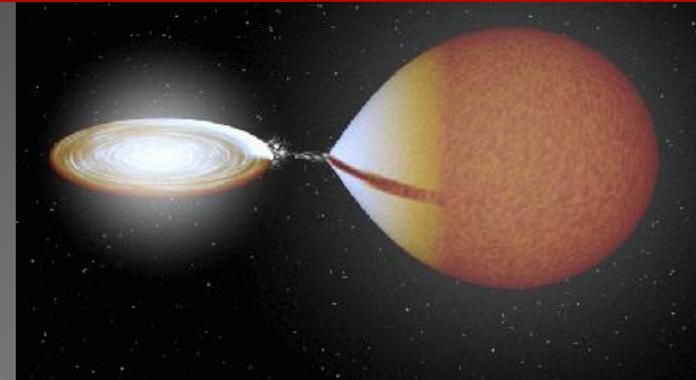
$$P_{\text{orb}} > P_{\text{bif}}$$

Diverging

→ LMXB widen their orbital period

Donor star is a (sub)giant

RLO driven by nuclear expansion



Formation of BMSPs with He-WD:

$$P_{\text{orb}} > 1 \text{ day}$$

$$0.18 < M_{\text{WD}} < 0.46 M_{\odot}$$

Unique relation between  $P_{\text{orb}}$  and  $M_{\text{WD}}$

Savonije (1987)

Joss, Rappaport & Lewis (1987)

Rappaport et al. (1995)

Tauris & Savonije (1999)

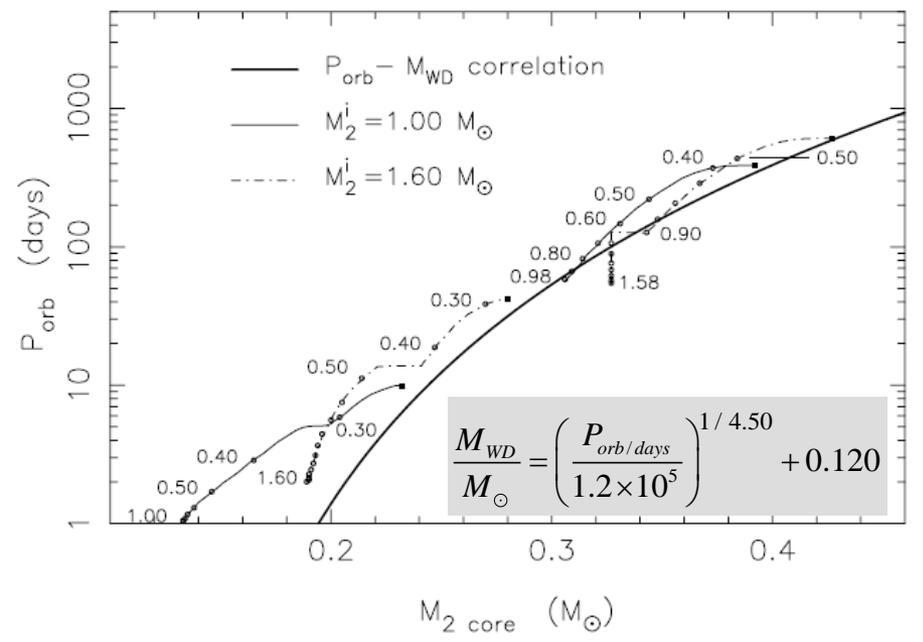
Istrate et al. (2016)



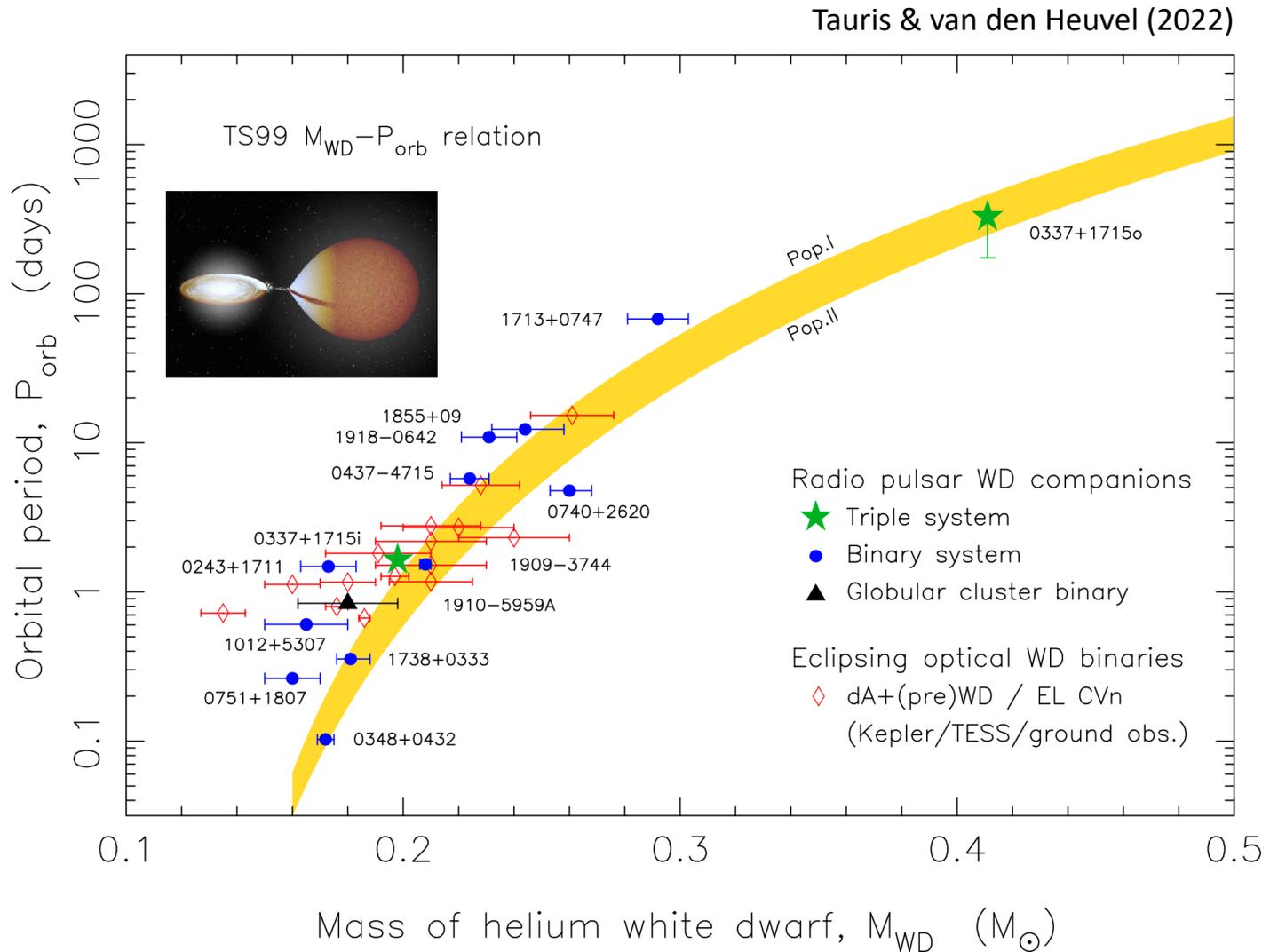
match observations well

938

T.M. Tauris & G.J. Savonije: Formation of millisecond

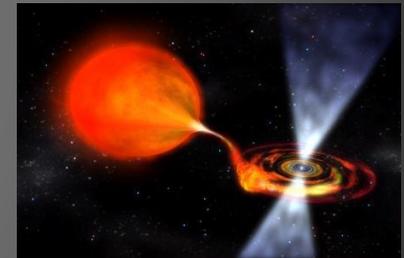
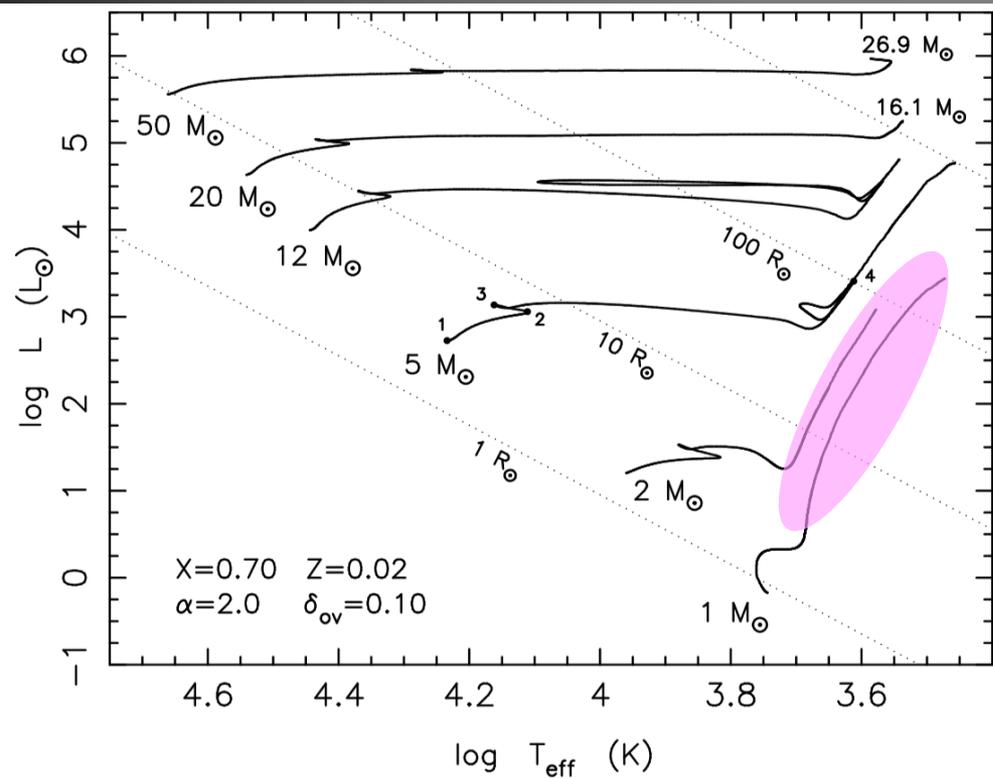


# $P_{\text{orb}} - M_{\text{WD}}$ correlation for He-WDs



# $P_{\text{orb}} - M_{\text{WD}}$ correlation for He-WDs

- On the red giant branch (hydrogen shell burning), the growth of the degenerate He core mass is directly related to the luminosity of the star
- Temperature is almost constant on the Hyashi track  $\Rightarrow L \propto R^2$   $L = 4\pi R^2 \sigma T_{\text{eff}}^4$
- Hence there is a relation between  $M_{\text{core}}$  and  $R$  (Thomas 1967) independent of  $M_{\text{env}}$
- The donor star fills its Roche-lobe during the mass transfer  $\Rightarrow R$  is correlated with  $P_{\text{orb}}$



## correlation between ( $P_{\text{orb}}$ , $M_{\text{WD}}$ )

**Table 1.** Stellar parameters for a star with  $R_2 = 50.0 R_{\odot}$  – see text.

$M_2/M_{\odot}$	1.0**	1.6**	1.0*	1.6*
$\log L/L_{\odot}$	2.566	2.624	2.644	2.723
$\log T_{\text{eff}}$	3.554	3.569	3.573	3.593
$M_{2\text{core}}/M_{\odot}$	0.336	0.345	0.342	0.354
$M_{2\text{env}}/M_{\odot}$	0.215	0.514	0.615	1.217

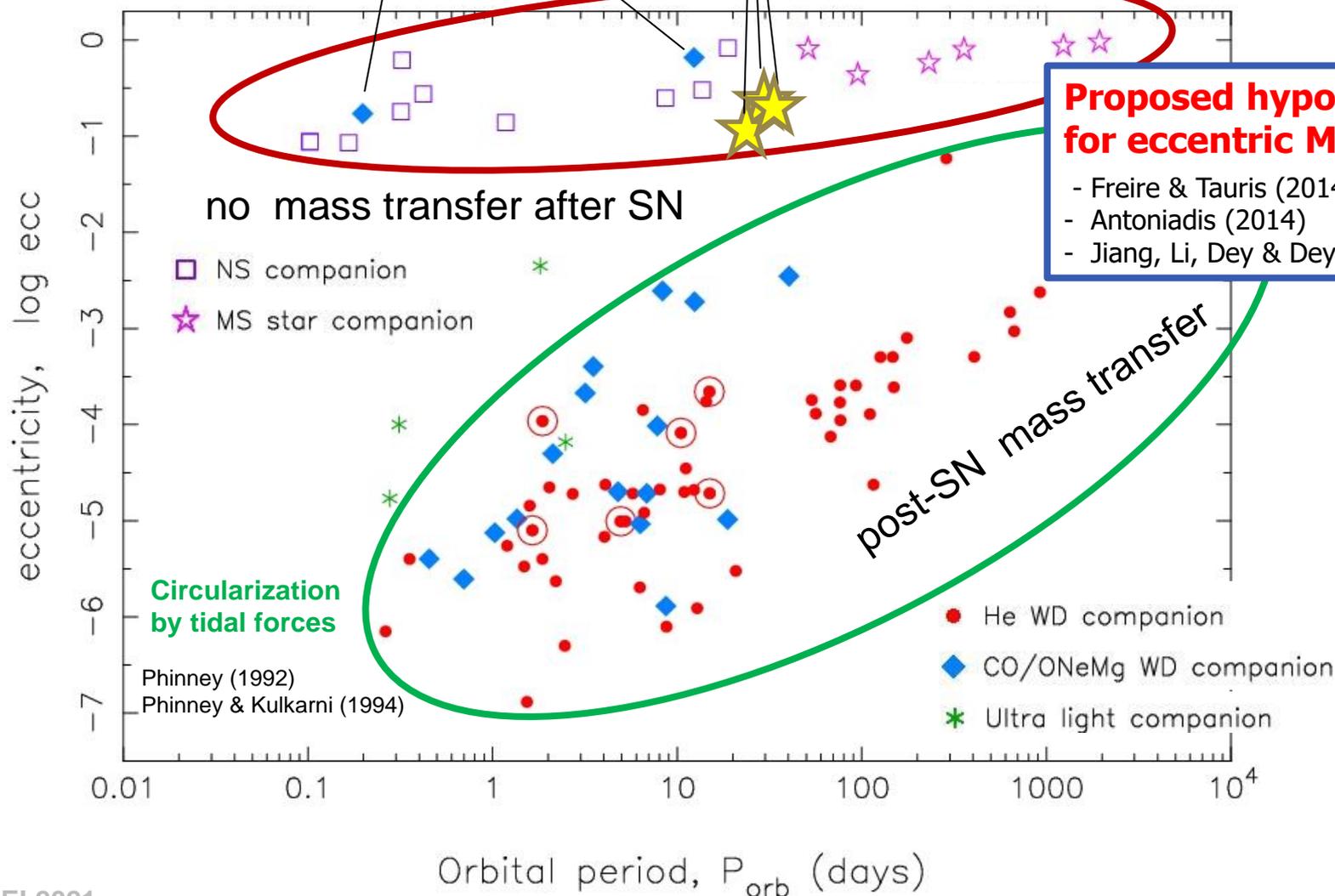
\* Single star ( $X=0.70$ ,  $Z=0.02$  and  $\alpha=2.0$ ).

\*\* Binary donor ( $P_{\text{orb}}^{\text{ZAMS}} = 60.0$  days and  $M_{\text{NS}} = 1.3 M_{\odot}$ )

# Eccentricities

**WDNS systems:** PSR B2303+46 (Tauris & Sennels, 2000) PSR J1141-6545

**Eccentric MSPs:** PSR J2234+06 (Deneva et al. 2013)  
 PSR J1946+3417 (Barr et al. 2013)  
 PSR J1950+2414 (Knispel et al. 2015)



**Proposed hypothesis for eccentric MSPs:**

- Freire & Tauris (2014)
- Antoniadis (2014)
- Jiang, Li, Dey & Dey (2015)

# Dynamical Effects of Asymmetric SNe

As a consequence of **sudden mass loss** and **imparted kicks in SNe**, radio pulsars have large velocities (0-1000 km s<sup>-1</sup>) and a wide scatter in their Galactic height distribution.

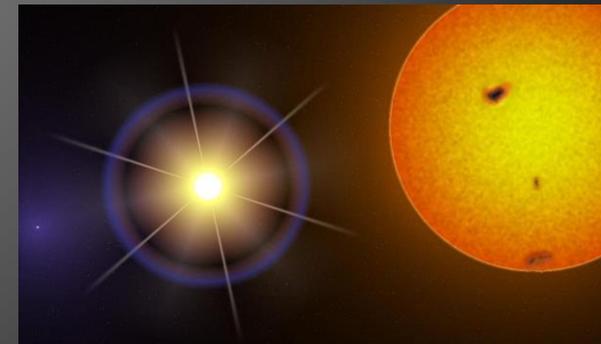
- Most (90%) of all potential LMXB systems are disrupted because of the SN.
- If SNe were purely symmetric then binary systems would be disrupted if  $\Delta M/M > 1/2$  (as a consequence of the virial theorem). Thus a kick can also help to keep systems bound if the direction of the kick is towards the companion star.

As a result of non-radial hydrodynamical instabilities newborn NSs receive a momentum kick at birth (resulting in a kick velocity of  $\sim 500$  km s<sup>-1</sup>).

The exact origin is still uncertain but is probably related to neutrino-driven convection bubbles, standing accretion shock instabilities (SASI) in the proto NS or other anisotropies in the ejecta which accelerate the proto-NS via the gravitational tug-boat mechanism, or simply an asymmetric neutrino outflow (see Janka 2012 for a review).

## Analytical equations for calculating dynamical effects of SNe:

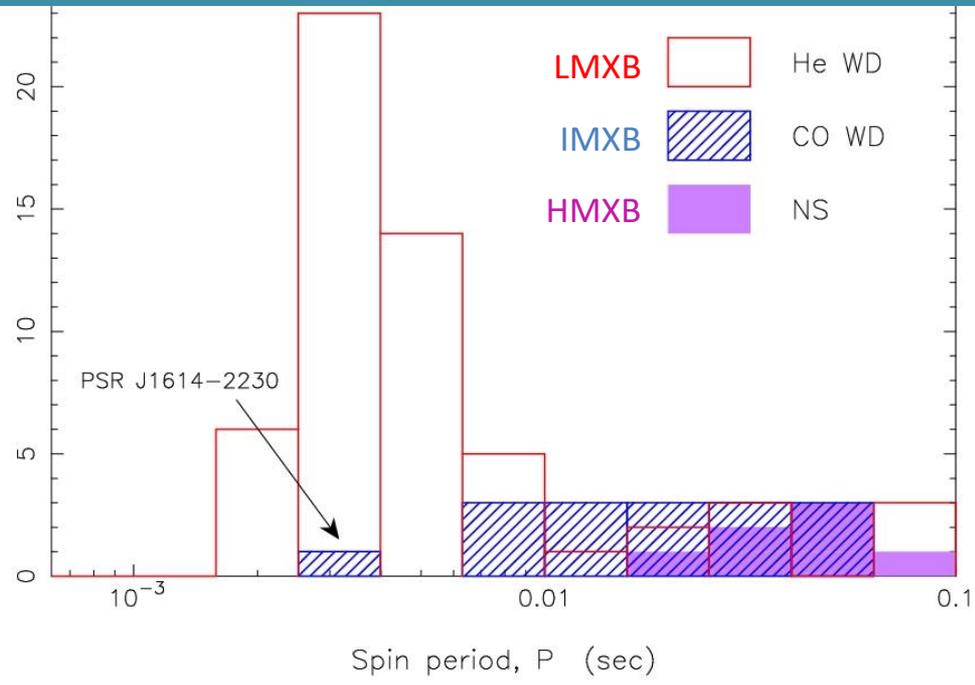
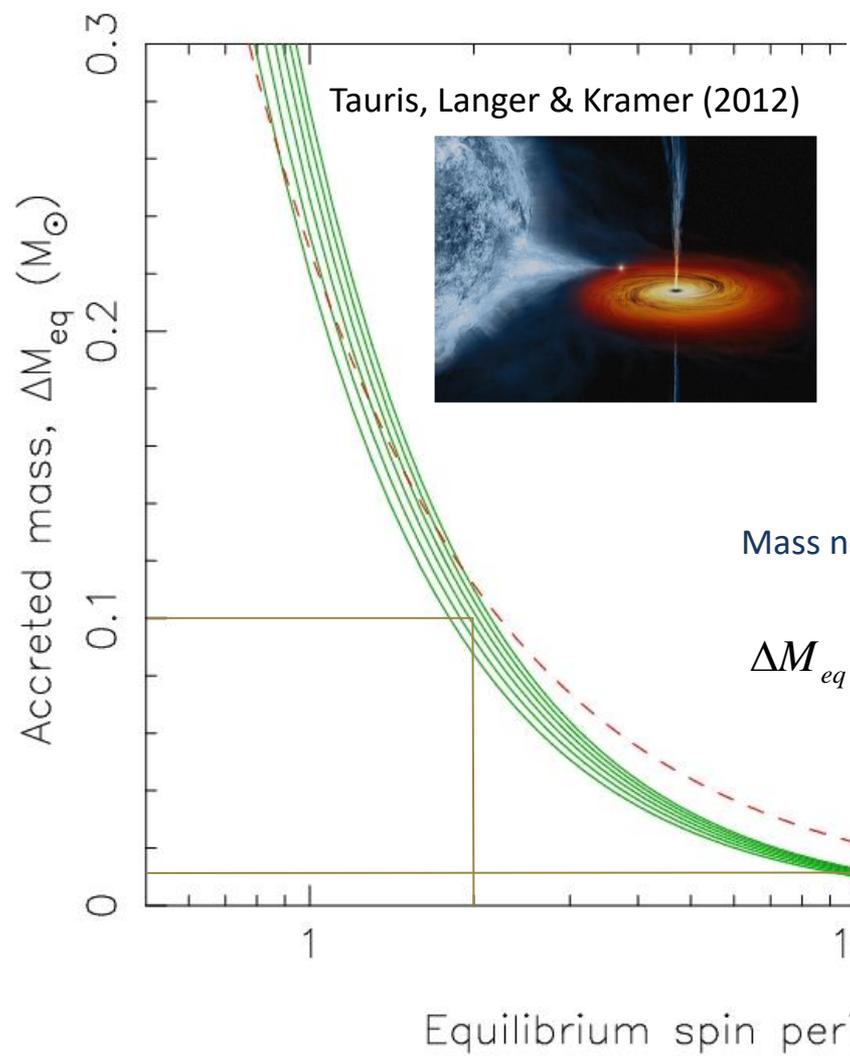
- Hills (1983) for binaries
- Tauris & Takens (1998) for the full general case (disrupted and bound binaries, incl. SN shell impact)



# Accreted mass to spin up pulsar

$$\Delta J_{\star} = \int n(\omega, t) \dot{M}(t) \sqrt{GM(t)r_{\text{mag}}(t)} \xi(t) dt$$

stellar evolution code



Mass needed to spin up pulsar:

$$\Delta M_{eq} \approx 0.22 M_{\odot} \frac{(M / M_{\odot})^{1/3}}{P_{ms}^{4/3}}$$

P (ms)	M (M <sub>sun</sub> )
0.7	0.40
2	0.10
5	0.03
10	0.01
50	0.001

# Spin-up line in the $P\dot{P}$ -diagram

$$\Omega_{NS} = \Omega_K(rm_{ag}) \quad \text{spin equilibrium}$$

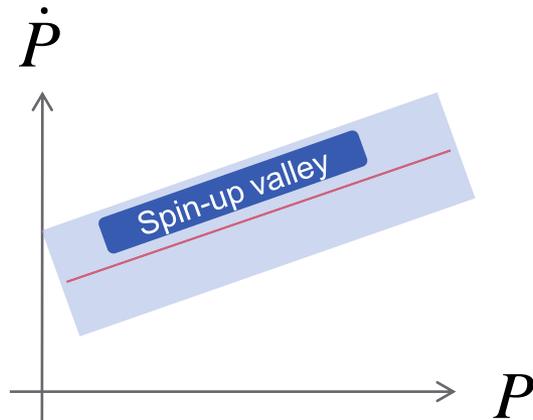
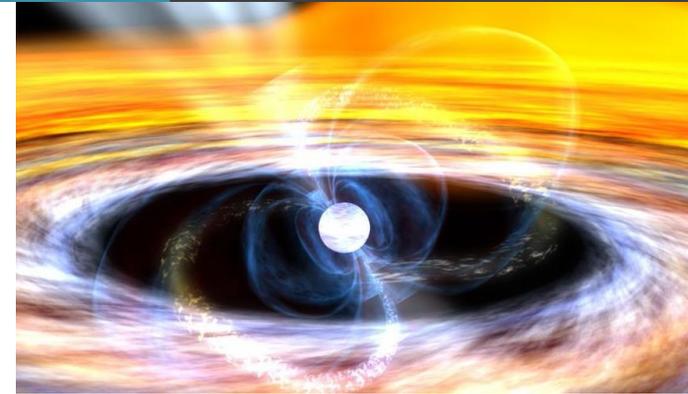
$$P_{eq} = 2\pi \sqrt{\frac{r_{mag}^3}{GM}} \frac{1}{\omega_c} \quad \wedge \quad r_{mag}(\dot{M}, B) \quad \wedge \quad B(P, \dot{P})$$

$$\dot{P} = \frac{2^{1/6} G^{5/3} \dot{M} M^{5/3} P_{eq}^{4/3}}{\pi^{1/3} c^3 I} \cdot (1 + \sin^2 \alpha) \cdot \varphi^{-7/2} \cdot \omega_c^{7/3}$$

spin-up line in  $P\dot{P}$  – diagram

Tauris, Langer & Kramer (2012)

Important!

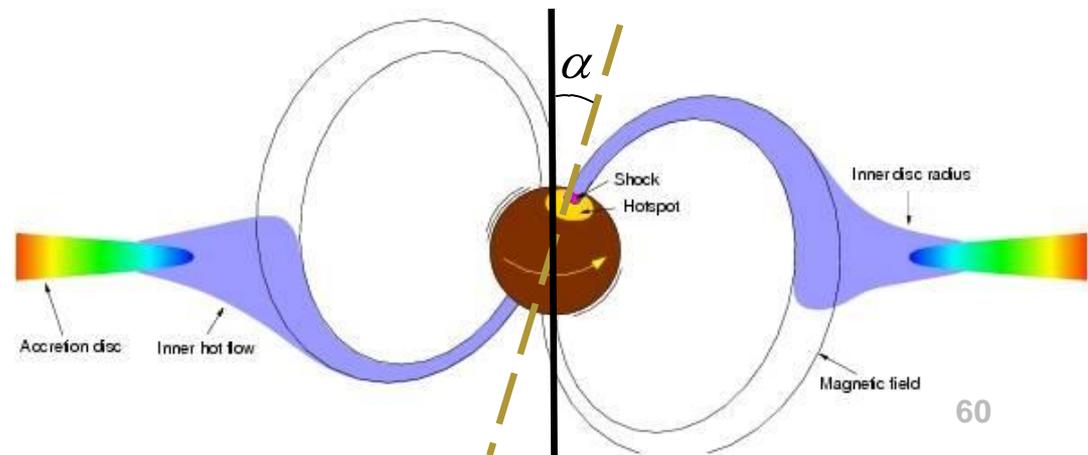


Classical spin-up line  
e.g. Bhattacharya & van den Heuvel (1991)

disk – magnetosphere parameters:

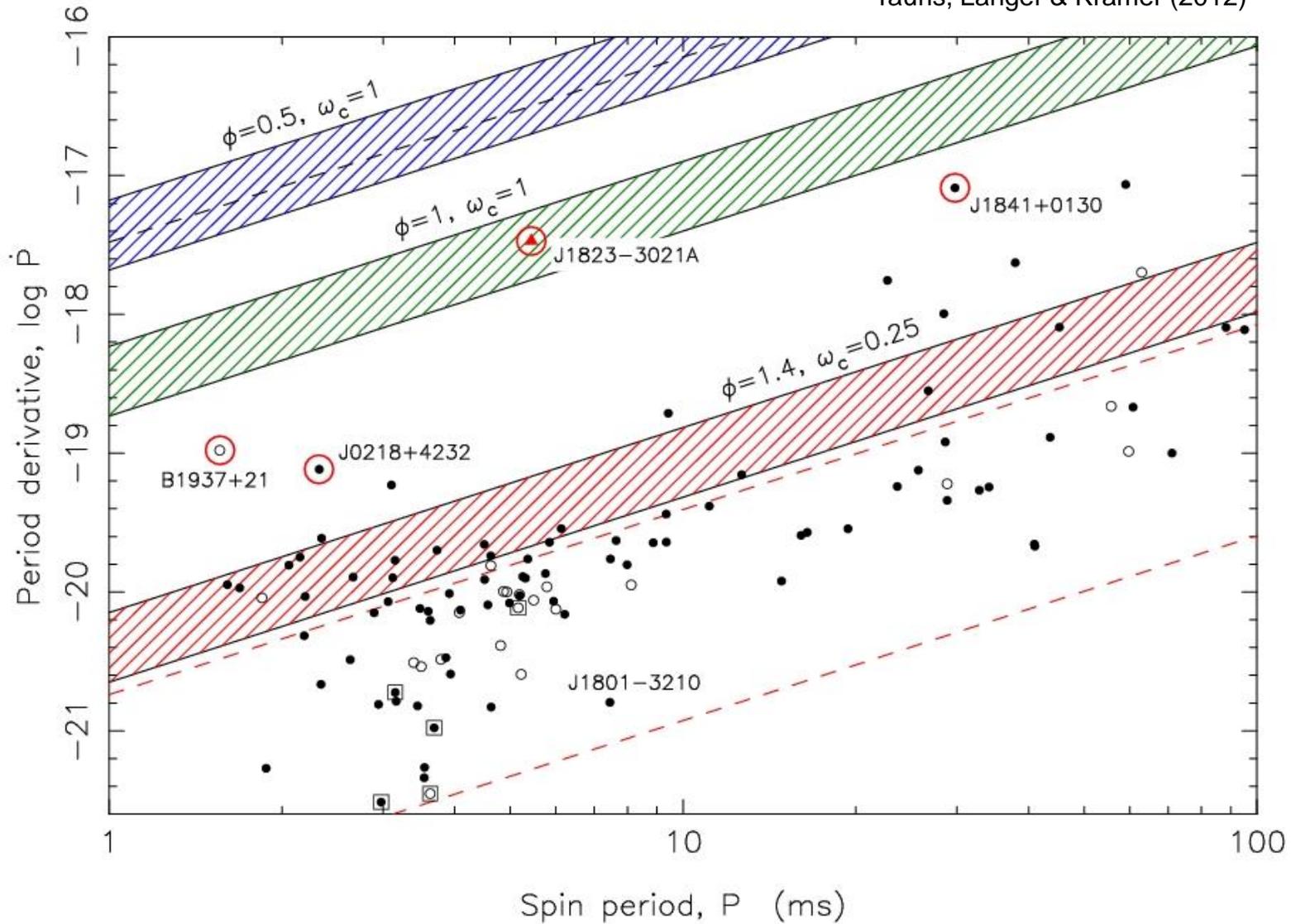
$$R_{mag} = \varphi R_{Alfven}$$

$$\Omega_{NS} = \omega_c \Omega_{mag}^{Kep.}$$

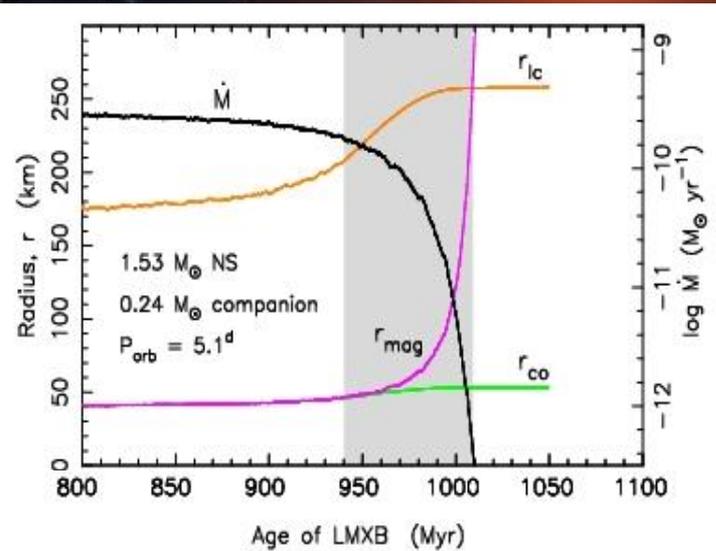


# Spin-up line

Tauris, Langer & Kramer (2012)



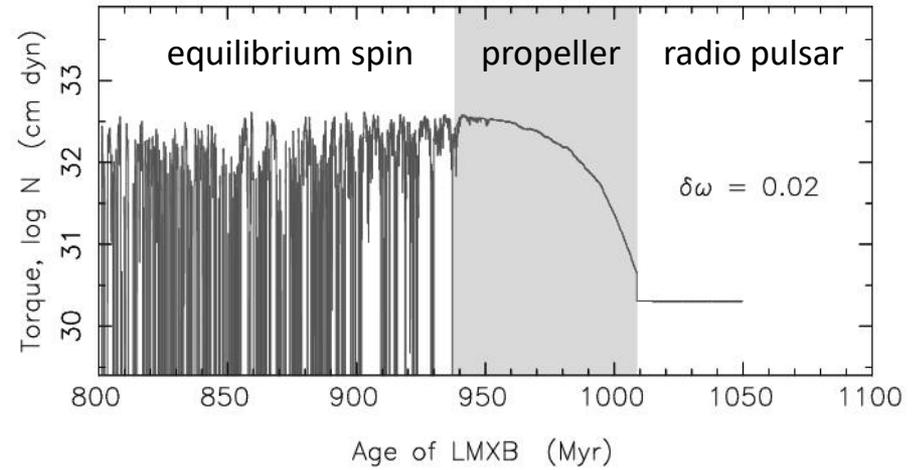
# Roche-lobe decoupling phase



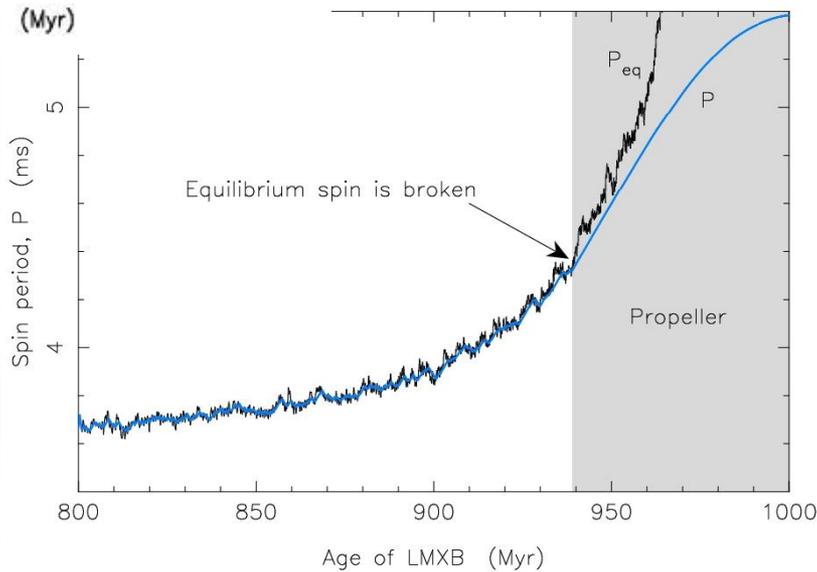
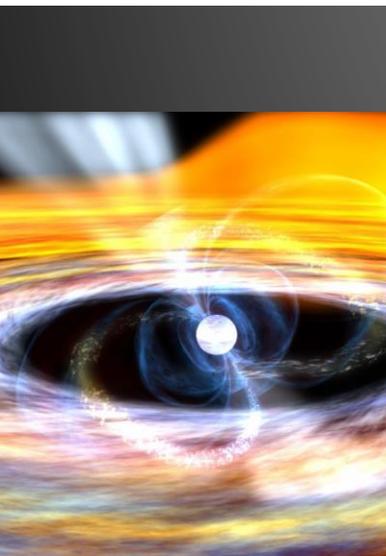
$$r_{mag} \approx r_{co}$$

$$r_{mag} > r_{co}$$

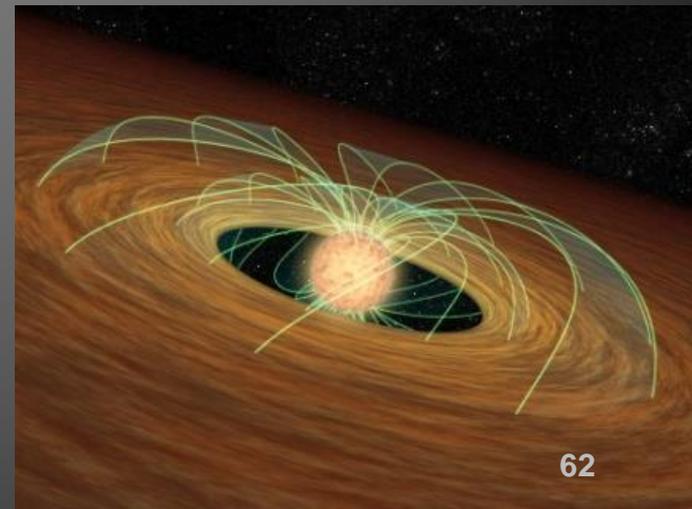
$$r_{mag} > r_{lc}$$



$$N(t) = n(\omega) \left[ \dot{M}(t) \sqrt{GM r_{mag}(t)} \xi + \frac{\mu^2}{9r_{mag}^3(t)} \right] - \frac{\dot{E}_{dipole}(t)}{\Omega(t)}$$



Tauris (2012), Science 335, 561



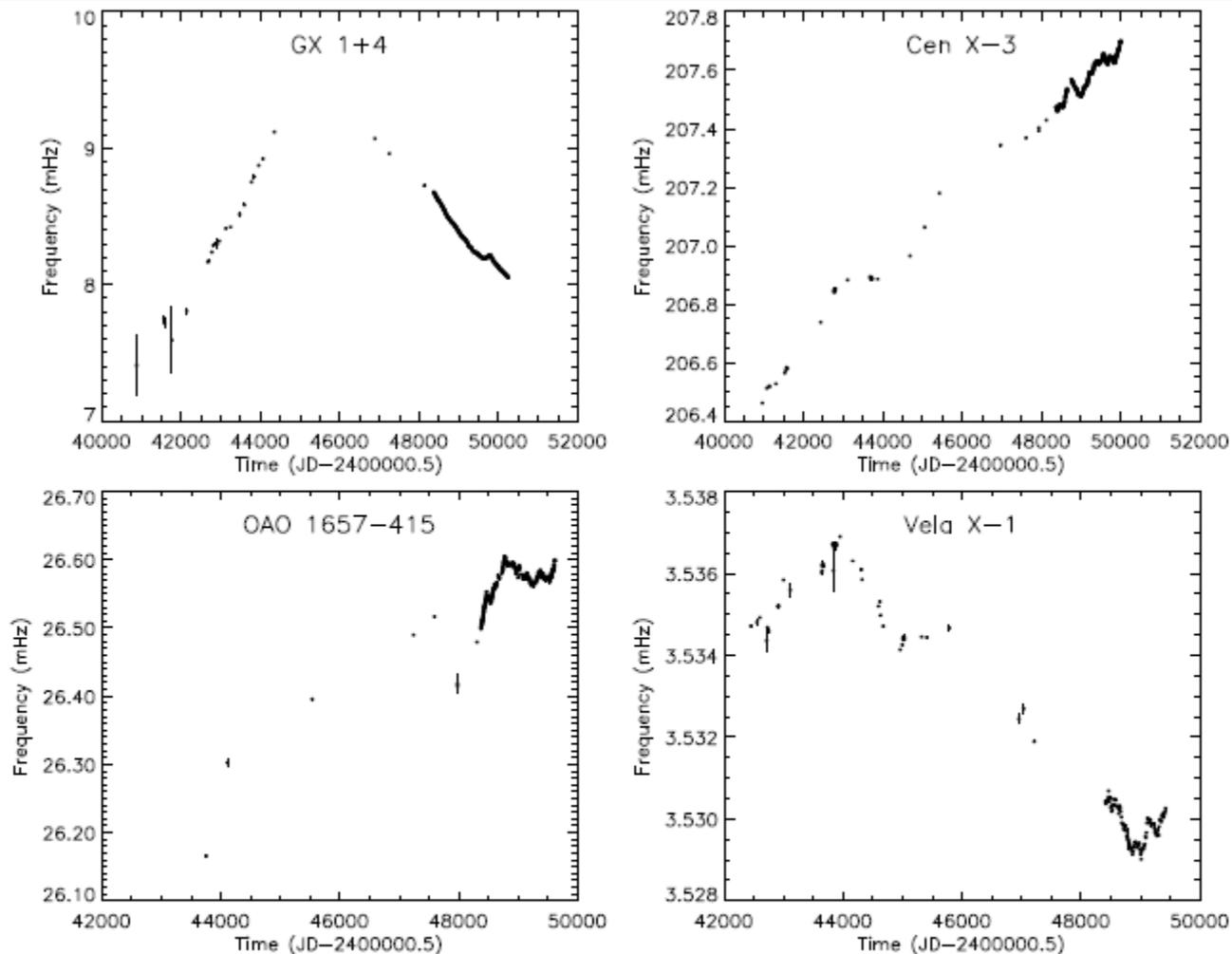


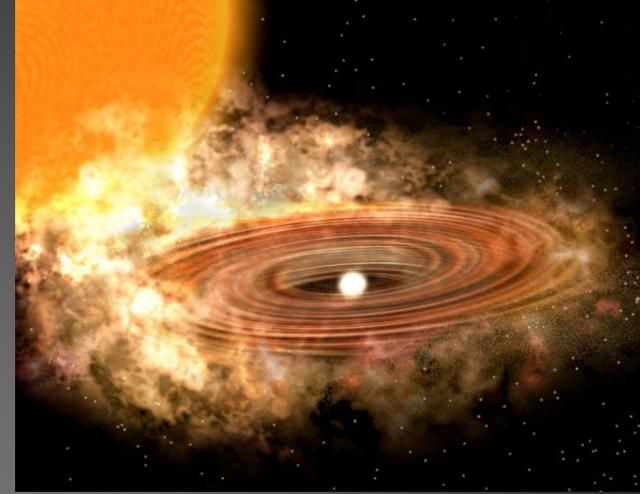
FIG. 6.—Long-term frequency history for all pulsars detected by BATSE that were previously known. The squares show the pre-BATSE data taken from Nagase (1989) and additional references. The line is the BATSE data, which we discuss later in great detail. The long-term frequency history for X-ray pulsars observed by BATSE that were known prior to the *Compton Observatory* launch commences 1991 April. For Her X-1, Cen X-3, Vela X-1, 4U 1538–52, GX 301–2, 4U 0115+634, and EXO 2030+375, all frequencies have been orbitally corrected. For OAO 1657–415, GS 0834–430, 2S 1417–62, and A0535+262, orbital corrections have been applied only to the BATSE observations. No orbital corrections have been applied for 4U 1626–67, GX 1+4, 4U 1145–619, or A1118–615, which have unknown, or incompletely known, orbital elements. The BATSE frequencies for OAO 1657–415, GS 0834–430,

Nagase (1989), Bildsten et al. (1997)

# Accretion Disks

High specific ang.mom. of accreted gas in binary

- formation of accretion disk  
(ang.mom. is transported outward via viscous stresses)



Turbulent-enhanced viscosity models (e.g.  $\alpha$ -model by Shakura & Sunyaev (1973))

- If accretion rate is  $< 0.01 \dot{M}_{\text{Edd}}$ : thin disk (high opacity) or ADAF (low opacity)
- If accretion rate is about  $\dot{M}_{\text{Edd}}$ : slim disks
- If accretion rate is  $> \dot{M}_{\text{Edd}}$ : torus (with collimated beam of radiation)

Magnetic stresses truncate the Keplerian disk flow:

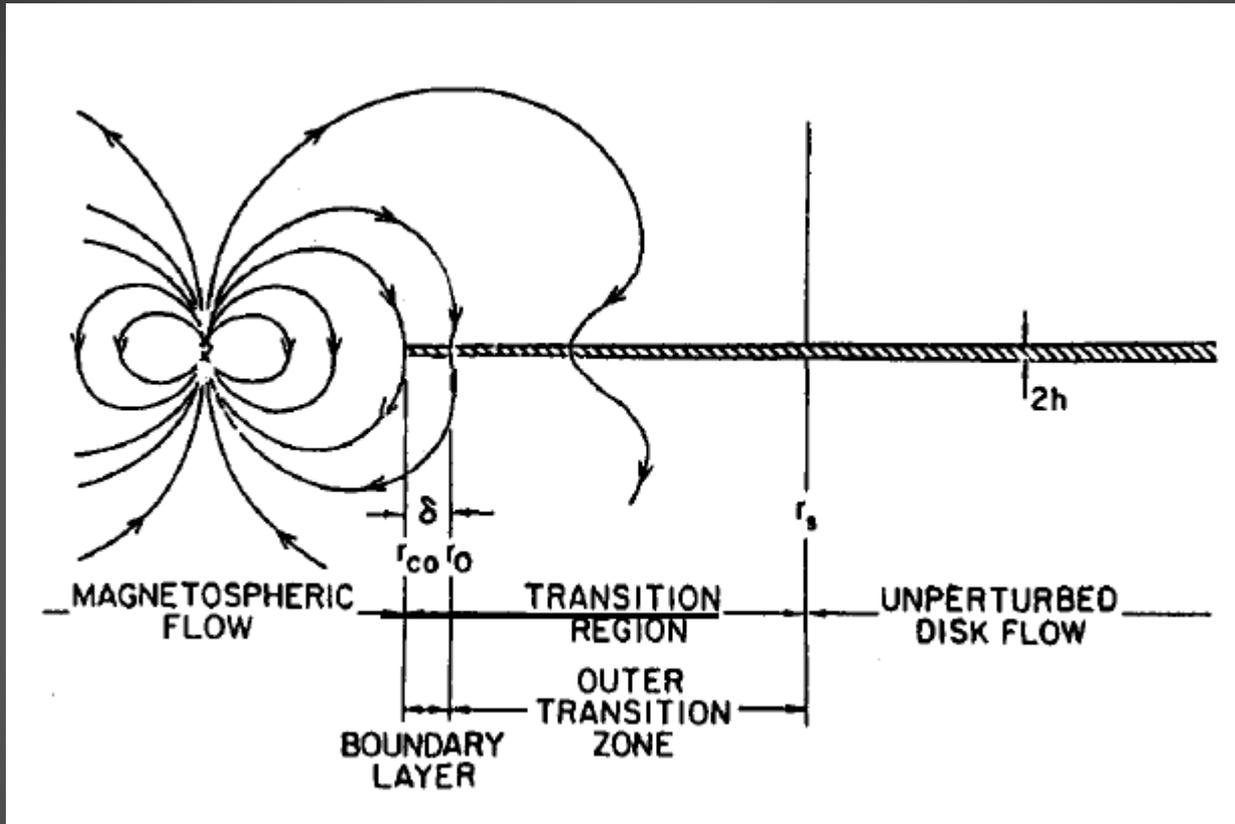
- transition zone between disk and magnetosphere

$$R_{\text{magnetosphere}} \propto R_{\text{inner disk}}$$

Spin-up lines in  $P$ - $\dot{P}_{\text{dot}}$  diagram depend on nature of accretion disk model  
(optically thick/thin and gas/radiation pressure dominated)

$$R_{\text{inner disk}} \propto \dot{M}^a \mu^b M^c$$





Ghosh & Lamb (1979)

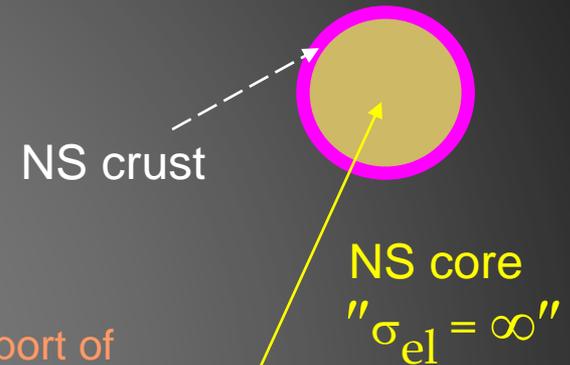
# Accretion-Induced Magnetic Field Decay

induction equation:

$$\frac{\partial \vec{B}}{\partial t} = - \underbrace{\frac{c^2}{4\pi} \vec{\nabla} \times \left( \frac{1}{\sigma_{el}} \times \vec{\nabla} \times \vec{B} \right)}_{\text{ohmic dissipation (diffusion)}} + \underbrace{\vec{\nabla} \times (\vec{v} \times \vec{B})}_{\text{convective transport of accreted material (Hall term)}}$$

ohmic dissipation (diffusion)

convective transport of accreted material (Hall term)



Bad approximation!

$$\frac{\partial B}{\partial t} = - \frac{c^2}{4\pi \sigma_{el}} \nabla^2 \vec{B} \Leftrightarrow B = B_0 e^{-t/\tau_D} \quad (\tau_D \approx \mu_0 \sigma_{el} L^2)$$

$$\sigma_{el} = \sigma_{el}(T, \rho, A, Z, Q)$$

Note: residual B-field ~10<sup>8</sup> G  
(observed in millisecond pulsars)  
due to superconducting interior

# Summary

- Introduction to X-ray binaries/Accretion
  - HMXBs and LMXBs
  - Roche-lobe overflow - Cases A, B and C
  - Stability criteria for mass transfer / stellar evolution
  - Orbital angular momentum balance equation
- Common envelope and spiral-in evolution
- Equilibrium spin period and spin-up line in  $P$ - $\dot{P}$  diagram
- Accretion physics
  - Accretion disks
  - B-field decay
  - Four phases of accretion (self study)

For a review: Tauris & van den Heuvel (2006)  
Tauris & van den Heuvel (2022)  
New textbook from Princeton Uni. Press



# Albert-Einstein Institute Lectures 2021

Thomas Tauris @ Aarhus University

Lectures 1+2: **Wednesday May 12, 10:00 – 12:00**

**X-ray Binaries and Recycling Millisecond Pulsars**

Lectures 3+4: **Friday May 14, 10:00 – 12:00**

**Spin and B-field Evolution of Neutron Stars (+ Black Hole Spins)**

Lectures 5+6: **Wednesday May 19, 10:00 – 12:00**

**Formation of Binary Neutron Stars/Black Holes**

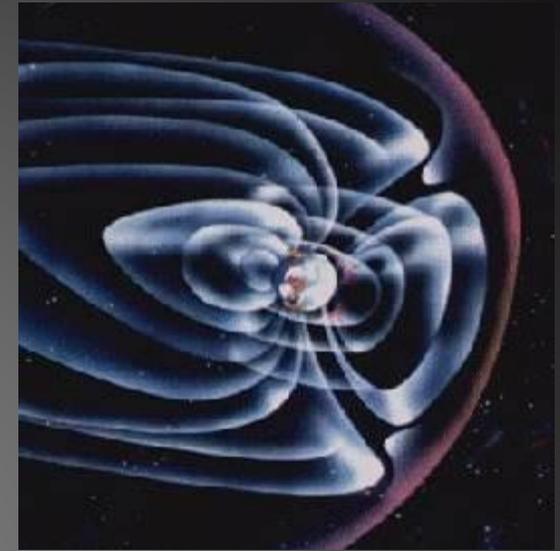
Lectures 7+8: **Friday May 21, 10:00 – 12:00**

**Binary Neutron Stars and Gravitational Waves at Low and High Frequencies**

**You are most welcome to ask questions any time 😊**

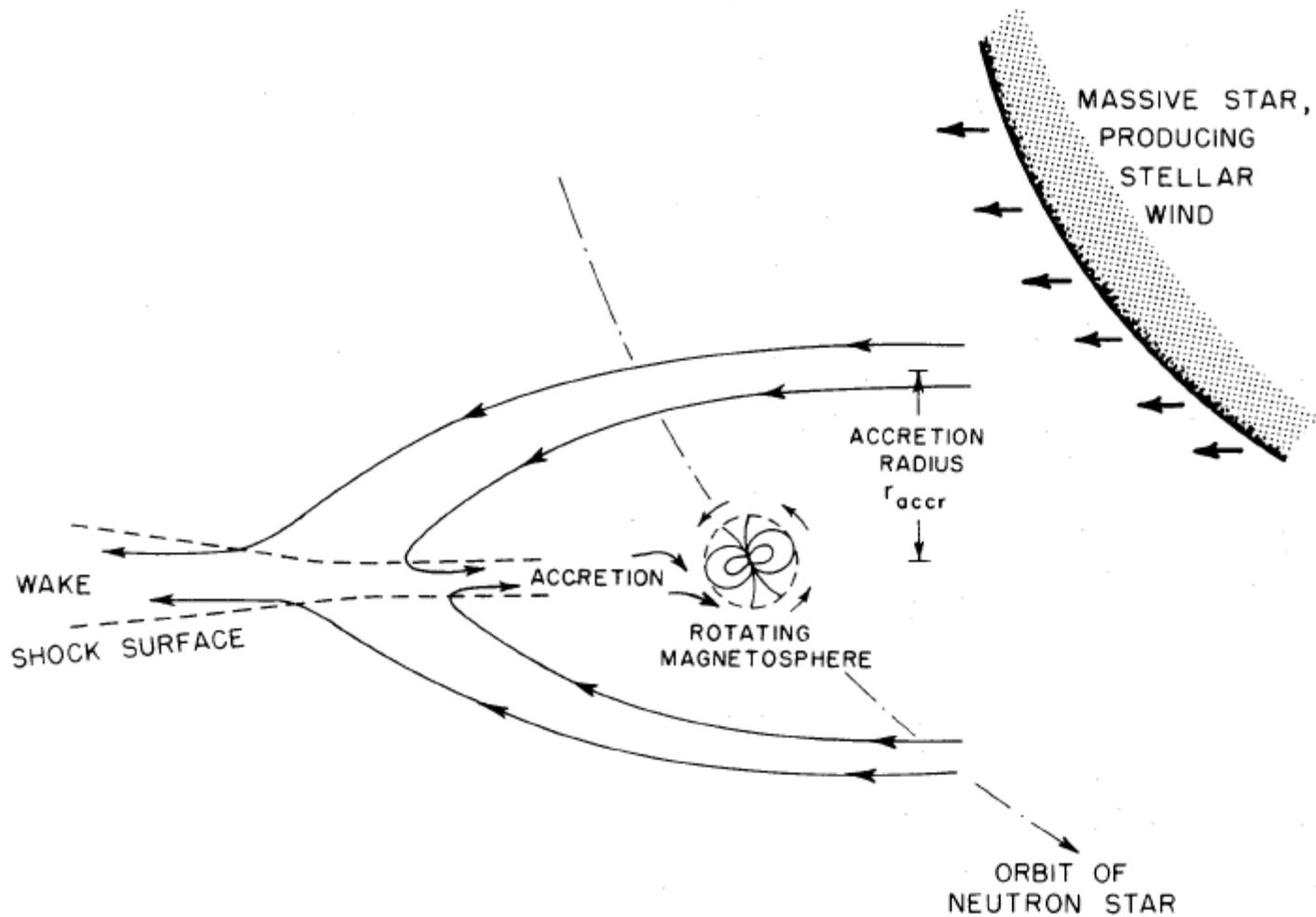
# Accretion Physics

Self study!



- ❖ Introducing the physics of an accreting neutron star
  - Spherical wind accretion
  - Effect of accretion disk

# NEUTRON-STAR ACCRETION



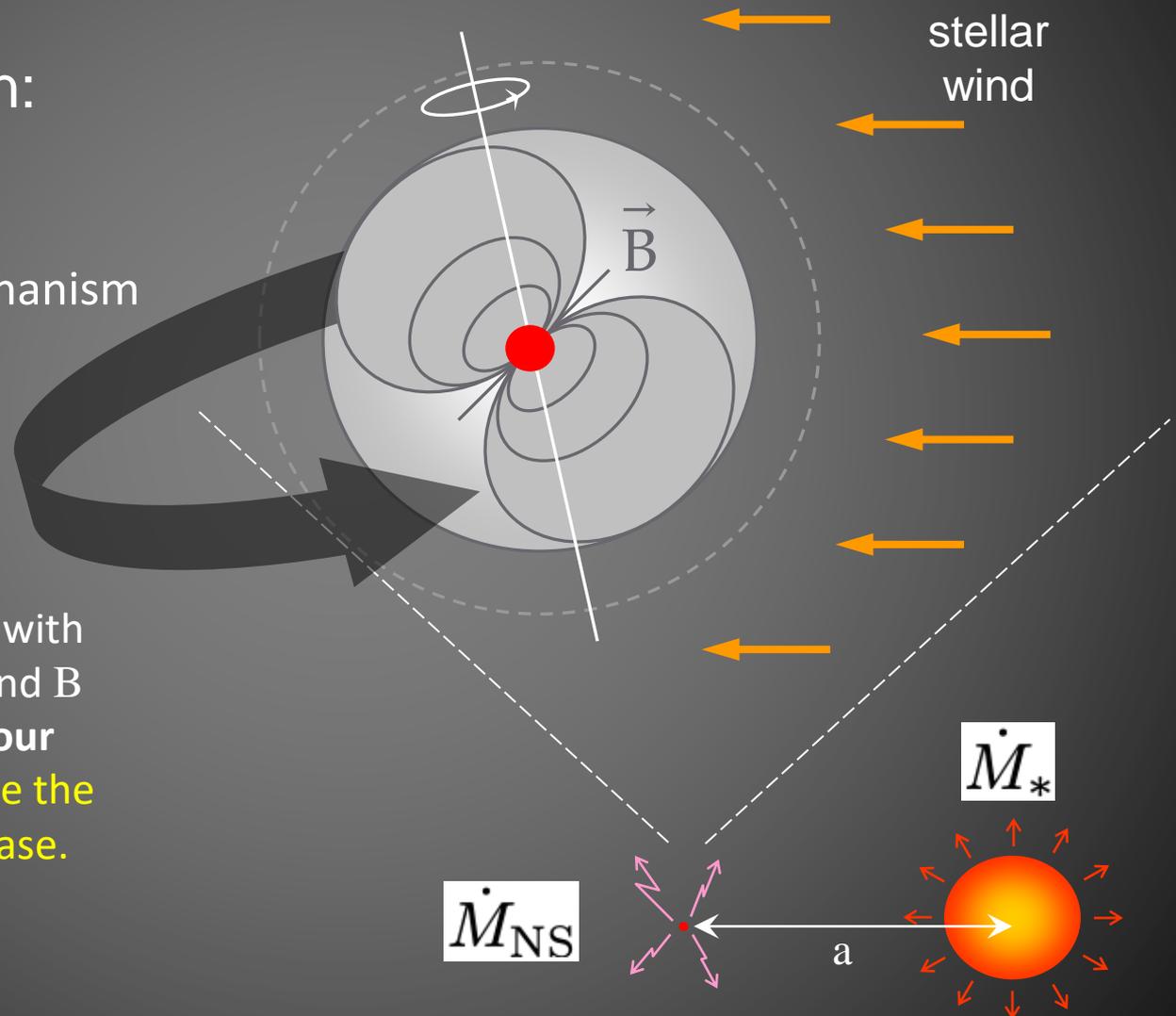
Davidson & Ostriker (1973)

# Neutron star accretion

## Phases of accretion:

- I. Isolated pulsar
- II. Gunn-Ostriker mechanism
- III. Propeller phase
- IV. Rapid accretion

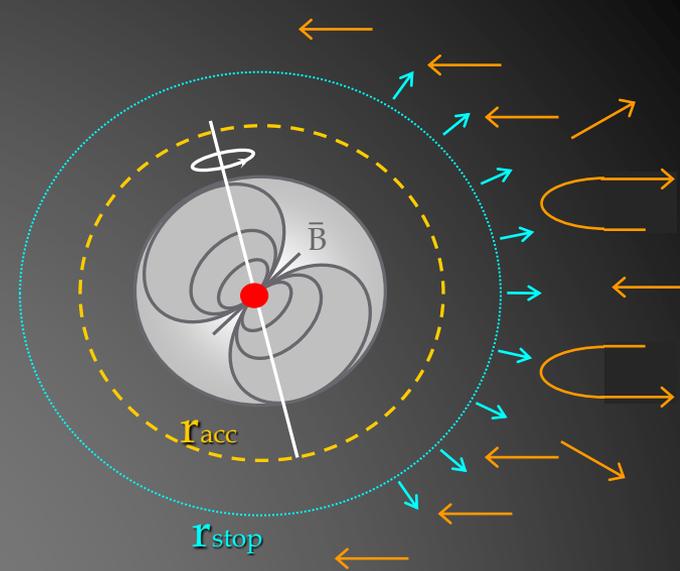
Consider a young pulsar with initial high values of  $\Omega$  and  $B$  which evolves through **four phases of accretion** while the values of  $\Omega$  and  $B$  decrease.



# Phase I

Isolated pulsar:  $r_{\text{stop}} > r_{\text{acc}}$

Wind plasma is stopped by pressure of magnetodipole radiation outside the radius of gravitational capture. The pulsar evolves as an isolated pulsar.



$$P_{\text{dipole}} \approx P_{\text{ram}}$$

$$\dot{E}_{\text{dipole}} = -\frac{2}{3c^3} |\ddot{m}|^2 \quad \wedge \quad |\ddot{m}| \sim BR^3\Omega^2$$

$$\frac{\dot{E}_{\text{dipole}}}{4\pi r_{\text{stop}}^2 c} = \frac{2B^2 R_{\text{NS}}^6 \Omega^4 / 3c^3}{4\pi r_{\text{stop}}^2 c} \approx \frac{1}{2} \rho_w v_w^2 = \frac{1}{2} \left( \frac{\dot{M}_*}{4\pi a^2 v_w} \right) v_w^2$$

continuity equation

$$\text{pressure} = \frac{\text{energy}}{\text{volume}}$$

ram pressure of wind

$$r_{\text{stop}} = \sqrt{\frac{4B^2 R_{\text{NS}}^6 \Omega^4 a^2}{3c^4 v_w \dot{M}_*}}$$

$$r_{\text{acc}} \sim \frac{2GM_{\text{NS}}}{v_w^2}$$

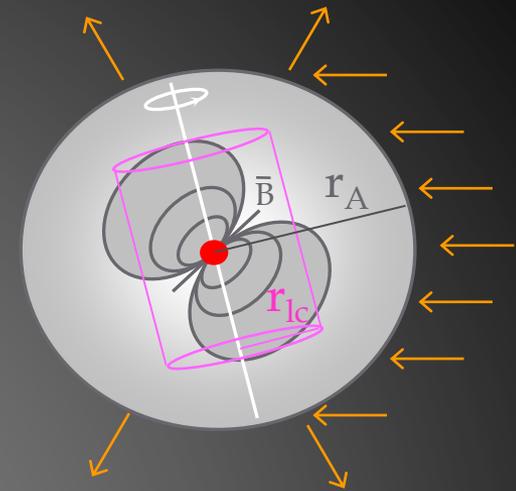
Over time, the pulsar loses rotational energy and  $\Omega$  decreases, causing  $r_{\text{stop}}$  to decrease... → Phase II

$$r_{\text{stop}} \propto B \Omega^2$$

# Phase II

Gunn-Ostriker mechanism:  $r_{\text{acc}}, r_A > r_{\text{stop}}, r_{\text{lc}}$

Now  $r_{\text{stop}} < r_{\text{acc}}$ . However, the Alfvén radius is located outside the light cylinder and matter cannot couple to the magnetosphere with  $v > c$ . Therefore, matter is accelerated to relativistic energies by magnetodipole waves.



$$\frac{B(r_A)^2}{8\pi} \approx \frac{1}{2} \rho_w v_{ff}^2$$

location of Alfvén radius

$$B(r_A) = B \left( \frac{R_{\text{NS}}}{r_A} \right)^3$$

perfect dipole

$$v_{ff} = \sqrt{\frac{2GM_{\text{NS}}}{r_A}}$$

approx. radial velocity

$$\rho_w = \frac{\dot{M}_*}{4\pi a^2 v_{ff}}$$

continuity equation

$$\dot{M}_{\text{NS}} = \frac{\pi r_A^2}{4\pi a^2} \dot{M}_*$$

accretion rate (solid angle accretion)

$$\Rightarrow r_A = \left( \frac{1}{32} \frac{B^4 R_{\text{NS}}^{12}}{GM_{\text{NS}} \dot{M}_{\text{NS}}^2} \right)^{1/7}$$

$$r_A \propto B^{4/7} \dot{M}_{\text{NS}}^{-2/7}$$

$$r_{\text{lc}} \equiv \frac{c}{\Omega}$$

location of light cylinder  
(distance from spin axis)

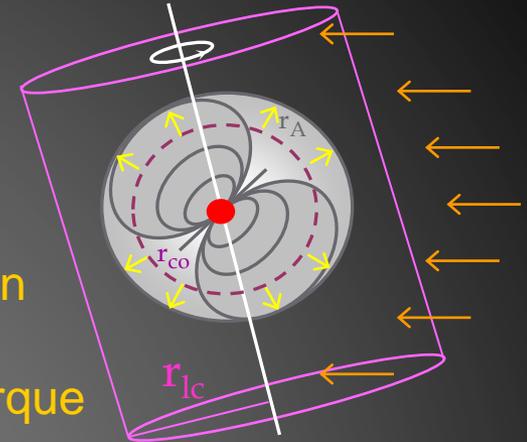
$$r_{\text{lc}} \propto \Omega^{-1}$$

Over time,  $\Omega$  decreases, causing  $r_{\text{lc}}$  to increase...  $\rightarrow$  Phase III

# Phase III

Propeller effect:  $r_{lc} > r_A > r_{co}$

Accreted matter couples to magnetosphere in super-Keplerian orbits ( $F_{centrifugal} > F_{gravitational}$ ) and thus material piles up near magnetospheric boundary, which creates a strong braking torque (wind carries off ang. mom.)



$$r_{co} = \left( \frac{GM_{NS}}{\Omega^2} \right)^{1/3}$$

co-rotation radius (Keplerian velocity)

$$N = \dot{J}_{spin} \approx \frac{\partial}{\partial t} (mr_A^2 \Omega_K) = \dot{M}_{NS} \sqrt{GM_{NS} r_A}$$

braking torque

$$\vec{J} = |\vec{r} \times \vec{p}|$$

$$\dot{\Omega} = \frac{\dot{J}_{spin}}{I_{NS}} \quad \wedge \quad \Omega = \frac{2\pi}{P} \quad \Rightarrow \quad \dot{P} \approx \frac{\dot{J}_{spin} P^2}{-2\pi I_{NS}} \propto L_X^{6/7}$$

spin-down rate

$$r_A^{1/2} \propto \dot{M}_{NS}^{-1/7}$$

$$L_X = \frac{dE_{acc}}{dt} = \frac{GM_{NS}}{R} \dot{M}_{NS} \propto \dot{M}_{NS}$$

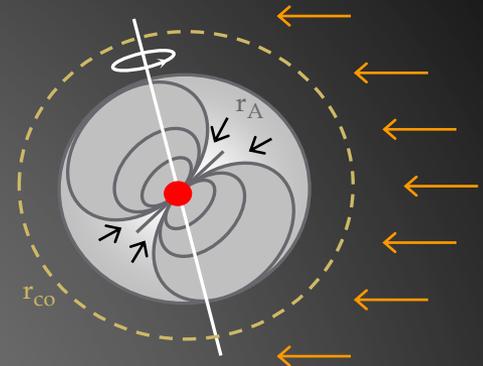
X-ray luminosity

$$r_{co} \propto \Omega^{-2/3}$$

Braking torque causes  $\Omega$  to decrease...  $\rightarrow$  Phase IV

# Phase IV

Neutron star accretion:  $r_A < r_{co}$



$$\dot{J}_{spin} = \dot{M}_{NS} \sqrt{GM_{NS} r_A} \xi$$

$\xi = 0.01 \sim 0.1$ , if no disk is formed (wind)  
 $\xi = 1$ , if accretion disk is formed (RLO)

$$\Omega_{NS} = \Omega_K(r = r_A) \iff \frac{2\pi}{P_{eq}} \equiv \sqrt{\frac{GM_{NS}}{r_A^3}}$$

equilibrium spin period!

$$P_{eq} \propto \dot{M}_{NS}^{-3/7} B^{6/7} R_{NS}^{18/7} M_{NS}^{-5/7}$$

See eq. for  $r_A$

see spin-up line in  $(P, \dot{P})$  diagram

Accretion is possible if  $P > P_{eq}$

In equilibrium  $r_{co}$  moves alternately inside and outside  $r_A$ .

