Problem set 2

1. Consider a merger of two black holes of arbitrary masses and spins. Suppose that the merger takes place at an *unknown* redshift z. Show that without knowing z, the waveform of the inspiral/merger/ringdown (meaning the observed frequencies, but not the amplitudes) is not sufficient to measure the masses or angular momenta of the black holes uniquely. What combinations of masses, spins, and redshifts can be measured?

2. This problem shows the limits of order of magnitude calculations in some cases. Let's say you'd like to estimate the recoil speed of a merged black hole remnant, due to linear momentum carried away by gravitational radiation. To simplify things, suppose we have two nonrotating black holes of masses M_1 and M_2 that collide head-on, so there is no spin at any point. A theorem from black hole thermodynamics says that the square of the irreducible mass of the final black hole cannot be less than the sum of the squares of the irreducible masses of the initial black holes. For nonrotating black holes, this becomes

$$M_{\rm final}^2 \geq M_1^2 + M_2^2 \ . \tag{1}$$

Like the increase in entropy, this is an *inequality*, but for our order of magnitude estimate we will assume $M_{\rm final}^2 = M_1^2 + M_2^2$.

With that assumption, compute the final speed of the remnant (as a fraction of the speed of light, and as a function of M_1 and M_2) assuming that all the radiated energy is carried away in a single direction. For comparison, the best current estimate is that the speed for $M_1/M_2 \approx 10$ is ~ 30 km s^{-1} for nonrotating holes that coalesce via a quasicircular inspiral.

3. Consider the ringdown produced by two $10\,M_\odot$ black holes. Suppose that the ringdown lasts for 2 cycles and emits a total of 1% of the mass-energy of the final black hole. Assuming a nonrotating black hole ($j = 0$), what would be the frequency of the radiation and how long would it last? The frequency is in the range of human hearing (although, of course, not audible!), and sound amplitude is measured in decibels, where 0 dB has an intensity of 10^{-9} erg cm^{-2} s^{-1}. If the BH-BH merger occurs at the distance of the Virgo Cluster (about 50 million light years, or 15 million parsecs, which is $\approx 5 \times 10^{25}$ cm), compare the intensity of the ringdown at Earth with the intensity of the loudest scream ever registered (129 dB, by Jill Drake of the UK). Then, do the same calculation for GW150914, which was the first event detected: two $\sim 30\ M_\odot$ black holes, final spin $j = 0.7$, distance 420 Mpc.

4. Compare the total energy ever emitted in gravitational waves with the total emitted in starlight. To do this, assume that gravitational wave energy is dominated by mergers of supermassive black holes (SMBH); that a typical current SMBH has 10^{-4} of the total

mass of the stars in a galaxy; and that in their lifetimes most SMBHs have one merger with a comparable-mass SMBH, and in doing so emit 5% of their mass-energy in gravitational waves. For the starlight, assume that within a Hubble time (the current age of the universe; you should round this to 10^{10} years) 5% of the mass in stars has converted from hydrogen to helium, with an efficiency of 0.7%; that is, the energy released has been $E = 0.007\Delta M$, where ΔM is 5% of the total stellar mass.