Problem set 1

1. Derive the constant of motion associated with inspiral according to the Peters equations. Hint: define $y \equiv e^2$ to get da/dt and dy/dt, then look for a constant in the form $C = af(y)$.

2. I claim in the notes that the frequency of a sound wave that involves most of a gravitationally bound object can't exceed $\sim (G\rho)^{1/2}$. But is that correct? Consider a spherical star of mass M and radius R. Recalling that the sound speed is $c_s = (dP/d\rho)^{1/2}$, where P is the pressure and ρ is the density, do a *simple* order of magnitude calculation to determine the sound speed and thus the crossing time of a sound wave across the star. The inverse of that time is of order the frequency. What are some objects that could potentially rotate or pulsate at a frequency much greater than $\sim (G\rho)^{1/2}$, and why?

3. A professor in College Park, Maryland, USA (which we will approximate as being 6500 km from Potsdam) waves his arms rapidly in the course of a lecture. His arm oscillation frequency is an amazing 100 Hz, and his waving is remarkably sinusoidal. Given plausible assumptions about the mass and radius of the professor's arms, could a detector in Potsdam detect his motion if its 1σ dimensionless strain sensitivity at 100 Hz is 5×10^{-23}?

4. Dr. I. M. N. Sane, noted astrophysical gadfly, has had a startling revelation: laser interferometers cannot possibly detect gravitational waves, and therefore the LVK collaboration has been fabricating data! He has realized that a gravitational wave will stretch or shrink *all* "measuring sticks" in the same way. One consequence of this is that if the length of an arm of a gravitational wave detector is changed by some fractional amount $\epsilon \ll 1$, then the wavelength of the laser light in the cavity is also changed by precisely that same factor. Therefore, the length of the cavity as measured in laser light wavelengths is unchanged by the gravitational wave, leading Dr. Sane to conclude that the interference patterns will be unchanged. Dr. Sane is considering this in the LIGO-like limit that the frequency of the gravitational wave is much less than c/L, where L is the unstretched length of an arm.

You have been approached by the US National Science Foundation to evaluate this argument. The future of gravitational wave detection depends on your response!